10/31/2024

Properties of Exponents and Logarithms

Review properties of exponents

$$a^m a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{m/n} = \sqrt[n]{a^m}$$

Properties of Logarithms

Definitional things:

$$a^x = y \ \leftrightarrow \log_a y = x$$

$$a^{\log_a x} = x$$
$$\log_{\mathrm{a}} a^x = x$$

Change of Base

$$\log_a b = \frac{\log b}{\log a} = \frac{\ln b}{\ln a}$$

Properties of Logarithms

$$\log_{\mathrm{a}}(MN) = \log_{\mathrm{a}}(M) + \log_{\mathrm{a}}(N)$$

$$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a x = -\log_{\frac{1}{a}} x$$

$$\log_a(M^r) = r \log_a M$$

Example.
Expand the log into several simpler log expressions.

$$\log(1000x^4y^5) =$$

$$\log 1000 + \log(x^4y^5) = \log 1000 + \log(x^4) + \log y^5 = \log(10^3) + \log(x^4) + \log(y^5) =$$

$$3 + 4\log x + 5\log y$$

Example.

$$\log_6\left(\frac{216}{x^3y}\right)^4 =$$

$$4\log_6\left(\frac{216}{x^3y}\right) = 4[\log_6 216 - \log_6 x^3y] = 4[\log_6 216 - (\log_6 x^3 + \log_6 y)] =$$

$$4[\log_6 216 - \log_6 x^3 - \log_6 y] = 4[\log_6 6^3 - 3\log_6 x - \log_6 y] =$$

$$4[3 - 3\log_6 x - \log_6 y] = 12 - 12\log_6 x - 4\log_6 y$$

$$\log_6\left(\frac{216}{x^3y}\right)^4 =$$

$$\log_6\left(\frac{6^3}{x^3y}\right)^4 = \log_6\frac{6^{12}}{x^{12}y^4} = \log_6 6^{12} - \log_6(x^{12}y^4) =$$

$$12 - (\log_6 x^{12} + \log_6 y^4) = 12 - \log_6 x^{12} - \log_6 y^4 =$$

$$12 - 12\log_6 x - 4\log_6 y$$

Example.

$$\log_2\left(\frac{4\sqrt[3]{x^2}}{y\sqrt{z}}\right) =$$

$$\log_2\left(\frac{4x^{2/3}}{yz^{1/2}}\right) = \log_2 4x^{2/3} - \log_2 yz^{1/2} = \log_2 4 + \log_2 x^{2/3} - \left(\log_2 y + \log_2 z^{1/2}\right) =$$

$$\log_2 2^2 + \frac{2}{3}\log_2 x - \log_2 y - \frac{1}{2}\log_2 z = 2 + \frac{2}{3}\log_2 x - \log_2 y - \frac{1}{2}\log_2 z$$

Combine the log expressions into a single log.
Example.

$$\log_2 x + \log_2 y - \log_2 z =$$

$$\log_2(xy) - \log_2 z = \log_2\left(\frac{xy}{z}\right)$$

Example.

$$\log(x) - \frac{1}{3}\log(z) + \frac{1}{2}\log y =$$

$$\log(x) - \log(z^{1/3}) + \log(y^{1/2}) = \log(x) + \log(y^{1/2}) - \log(z^{1/3})$$

$$\log\left(\frac{x}{z^{1/3}}\right) + \log(y^{1/2}) = \log(xy^{1/2}) - \log(z^{1/3})$$

$$\log\left(\frac{xy^{1/2}}{z^{1/3}}\right) = \log\left(\frac{xy^{1/2}}{z^{1/3}}\right)$$

$$\log\left(\frac{x\sqrt{y}}{\sqrt[3]{z}}\right)$$

Example.

$$\log_7 x + \log_7(x-3) - 2 =$$

$$\log_7(x(x-3)) - 2 = \log_7(x(x-3)) - \log_7 7^2 = \log_7(x(x-3)) - \log_7 49 =$$

$$\log_7\left(\frac{x(x-3)}{49}\right)$$

42. Give numerical examples to show that, in general,

(a) $\log_b(x+y) \neq \log_b(x) + \log_b(y)$

(b) $\log_b(x-y) \neq \log_b(x) - \log_b(y)$

(c) $\log_b\left(\frac{x}{y}\right) \neq \frac{\log_b(x)}{\log_b(y)}$

Next week: solving equations with logs and exponentials

Election Day is a holiday, no official classes

Check announcements regularly before Tuesday class from now on.