MTH 161, Quiz #6, Fall 2024 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Simplify, and write in standard form.
   a. $(4-8i)(-3+i)$

(-1)

$-12+4i+24i-8i^2$

$-12+4i+24i+8 = -4+28i$

   b. $\frac{3+2i}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{12+9i+8i+6i^2}{16+9} = \frac{6+17i}{25}$ (-1)

$\frac{6}{25}+\frac{17i}{25}$

2. One zero of the polynomial equation $x^4-2x^2-16x-15=0$ is $x=3$. Use polynomial division to reduce the polynomial. Then find the rest of the real and complex zeros of the function. You may use the Rational Zero's Theorem and/or The Remainder Theorem. Write the final factored form of the polynomial with linear factors or quadratics with real coefficients (when the roots are complex). Graph the function.

x^3+3x^2+7x+5

$x-3 \,)\, x^4-0x^3-2x^2-16x-15$

$-x^4+3x^3$

$3x^3-2x^2$

$-3x^3+9x^2$

$7x^2-16x$

$-7x^2+21x$

$5x-15$

Rational zeros: $\pm1, \pm5$

x^2+2x+5

$x+1 \,)\, x^3+3x^2+7x+5$

$-x^3 \mp x^2$

$2x^2+7x$

$-2x^2 \mp 2x$

$5x+5$

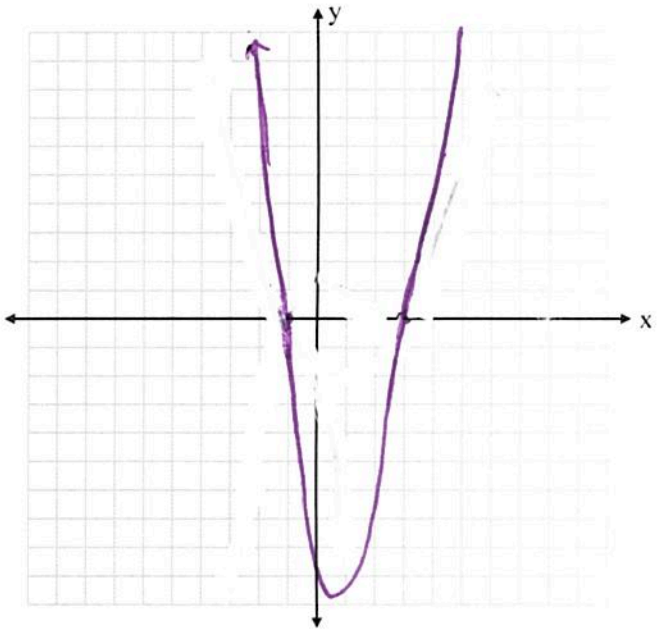
$5x+5$

0

$x^2+2x+5=0$

$x=\frac{-2\pm\sqrt{4-4(5)}}{2}=\frac{-2\pm 4i}{2}=-1\pm2i$

$(x-3)(x+1)(x^2+2x+5)$



Zeros: $3, -1, -1\pm2i$

3. Find all the real and complex zeros of the polynomial $p(x) = x^5 - x^4 + 7x^3 - 7x^2 + 12x - 12$.

$x^4(x-1)+7x^2(x-1)+12(x-1)$

$(x^4+7x^2+12)(x-1)$

$(x^2+4)(x^2+3)(x-1)$

$x = \pm 2i, \pm\sqrt{3}i, x=1$

4. Create a polynomial with the following properties:
    - The solutions to $f(x) = 0$ are $x = \pm 2, x = \pm 7i$
    - The leading term of $f(x)$ is $-3x^5$
    - The point (2,0) is a local maximum on the graph of $y = f(x)$.

(2,0)

$(x-2)(x+2)(x-7i)(x+7i)$

$a(x^2-4)(x^2+49)$

$a(x^4+49x^2-4x^2-196)$

$a(x^4+45x^2-196)$

$-3x^4-135x^2+588 = f(x)$

$(x-2)(-3x^4-135x^2+588)$

$-3x^5-135x^3+588x+6x^4+270x^2-1176$

$-3x^5+6x^4-135x^3+270x^2+588x-1176 = f(x)$