Single Variable Differentiation Review Key

1) $f(t) = 3t^4 + 6t^{1/3} - \frac{1}{2}\cdot\frac{1}{t}$

$f'(t) = 12t^3 + 2t^{-2/3} + \frac{1}{2}t^{-2} = 12t^3 + \frac{2}{\sqrt[3]{t^2}} + \frac{1}{2t^2}$

2) $g'(s) = \cos(s) + 20\sin(5s)$

3) $h'(t) = 4e^{4t} + e^{-t}$

4) $f(x) = 3x^3 + 6x^2 + 7x + 14$

$f'(x) = 9x^2 + 12x + 7$

5) $g'(t) = \frac{(2t+1)(t^2-1) - (2t)(t^2+t-1)}{(t^2-1)^2} = \frac{2t^3+t^2-2t-1-(2t^3+2t^2-2t)}{(t^2-1)^2}$

$= \frac{\cancel{2t^3}+t^2-\cancel{2t}-1-\cancel{2t^3}-2t^2+\cancel{2t}}{(t^2-1)^2} = \frac{-t^2-1}{(t^2-1)^2}$

6) $h(x) = x^{1/2}\sin x$

$h'(x) = \frac{1}{2}x^{-1/2}\sin x + x^{1/2}\cos x = \frac{\sin x}{2x^{1/2}} + x^{1/2}\cos x\,\frac{2x^{1/2}}{2x^{1/2}} =$

$\frac{\sin x + 2x\cos x}{2\sqrt{x}}$

7) $y'(t) = 4e^t + 4te^t = 4e^t(1+t)$

8) $g(x) = (1-x^3)^{1/2}$

$g'(x) = 1/2(1-x^3)^{-1/2}(-3x^2) = \frac{-3x^2}{2\sqrt{1-x^3}}$

9) $f'(s) = 5\left(s^2+\frac{1}{s}\right)^4\left(2s-\frac{1}{s^2}\right) = 5\left[\frac{1}{s}(s^3+1)\right]^4\frac{1}{s^2}(2s^3-1) = \frac{5(2s^3-1)(s^3+1)^4}{s^6}$

10) $y'(\theta) = -3\csc(3\theta)\cot(3\theta) - 4\theta\csc^2(2\theta^2)$

11) $y(x) = 2(\cosh 2x)^{1/2}$

$y'(x) = \cancel{2}\cdot\frac{1}{\cancel{2}}(\cosh 2x)^{-1/2}\cdot 2\sinh 2x = \dfrac{2\sinh(2x)}{\sqrt{\cosh(2x)}}$

12) $s(t) = \frac{1}{2}\ln(1+t^2)$

$s'(t) = \frac{1}{\cancel{2}}\cdot\frac{1}{1+t^2}\cdot\cancel{2}t = \dfrac{t}{1+t^2}$

13) $h'(z) = \frac{1}{\cancel{2}}e^{\sin(2z)}\cos(2z)\cdot\cancel{2} = e^{\sin(2z)}\cos(2z)$

14) $g(x) = x(\ln x)^{1/3}$

$g'(x) = 1\cdot(\ln x)^{1/3} + \cancel{x}\cdot\frac{1}{3}(\ln x)^{-2/3}\cdot\frac{1}{\cancel{x}} = (\ln x)^{1/3} + \frac{1}{3(\ln x)^{2/3}} =$

$$\frac{3\ln x + 1}{3\sqrt[3]{(\ln x)^2}}$$

15) $f'(y) = 6\sec^2(y^2)\cdot\sec(y^2)\tan(y^2)\cdot 2y =$

$12y\sec^3(y^2)\tan(y^2)$

16) $g'(y) = \dfrac{1}{1+\left(\frac{y}{a}\right)^2}\cdot\frac{1}{a}\cdot\frac{a}{a} = \dfrac{a}{a^2+y^2}$

17) $a'(x) = \arcsin^2(x) + x\,2\arcsin(x)\cdot\dfrac{1}{\sqrt{1-x^2}} - 2 + \cancel{2}(\tfrac{1}{\cancel{2}})(1-x^2)^{-1/2}(-2x)\arcsin x$

$+ 2\cancel{\sqrt{1-x^2}}\cdot\dfrac{1}{\cancel{\sqrt{1-x^2}}}$

$= \arcsin^2 x + \cancel{\dfrac{2x\arcsin x}{\sqrt{1-x^2}}} \cancel{-2} - \cancel{\dfrac{2x\arcsin x}{\sqrt{1-x^2}}} \cancel{+2} = \arcsin^2 x$

18) $g'(y) = \dfrac{1}{\sinh(y)} \cdot \cosh y = \coth y$

19) $f'(q) = 4q^3 \sin(\sqrt{3}\,q) + q^4 \cos(\sqrt{3}\,q) \cdot \sqrt{3} =$

$q^3\left[4 \sin(\sqrt{3}\,q) + \sqrt{3}\,q \cos(\sqrt{3}\,q)\right]$

20) $F'(t) = \dfrac{1}{1+\tanh^2 t} \cdot \operatorname{sech}^2 t - \operatorname{sech} t + t \operatorname{sech} t \tanh t$

$= \dfrac{\operatorname{sech}^2 t}{1+\tanh^2 t} - \operatorname{sech} t + t \operatorname{sech} t \tanh t$

21) $Y'(x) = \dfrac{1}{\sec x + \tan x} \cdot \sec x \tan x + \sec^2 x = \dfrac{\sec x(\cancel{\tan x + \sec x})}{\cancel{\sec x + \tan x}} = \sec x$

22) $\ln(G(x)) = \ln[x^{\ln x}] = (\ln x)^2$

$\dfrac{1}{G(x)} \cdot G'(x) = 2 \ln x \cdot \dfrac{1}{x} \Rightarrow G'(x) = \dfrac{2\ln x}{x} \cdot G(x)$

$= \dfrac{2 \ln x}{x} \cdot x^{\ln x}$

23) $H'(x) = x^2 \sqrt[3]{1-x^3}$ (by second fundamental theorem of calculus)

24) $T'(t) = e^{\sin^3(t^2)} \cdot \cos(t^2) \cdot 2t = 2t\, e^{\sin^3(t^2)} \cos(t^2)$ (ditto)

25) $p'(t) = 3^{4t} \cdot \ln(3) \cdot 4 = (4 \ln 3) \cdot 3^{4t}$