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Sets of Numbers

Numbers come with all kinds of properties, and so mathematicians divide numbers into sets where all the numbers have similar properties. On the last page is a chart of all the different number sets you will encounter in algebra. (The set of Complex Numbers will be discussed in Intermediate Algebra, but the others are discussed in Beginning Algebra.) We want to know what kinds of numbers belong in each set. I will describe them below, starting from the bottom of the tree. The sets in **bold** are the special sets you need to know.

Natural Numbers: This set of numbers is also called the set of Counting Numbers. Where do you begin counting? At 1 of course! 1, 2, 3, etc. All whole, positive numbers belong in this set. This set used so often in mathematics that it has a special symbol: □ .

Whole Numbers: This set is just the set of Natural Numbers + the number Zero.

Integers: This set of numbers contains all the positive whole numbers, 0 and the negative whole numbers. So included in the set are things like -4, -16, -85 and also 0, 5, 10, 17, etc. This set is also special enough to get its own symbol: □ . The Z is for the German word for number "Zimmer" since the N was already taken for the Natural Numbers, and I is taken elsewhere.

Rational Numbers: This set of numbers includes all numbers that can be represented by fractions where the denominator is not zero. Since we can write all integers over 1 and get an equivalent number, these are all here, as well as familiar proper, improper and mixed fractions: 1/1, ¾, 5/2, etc. This set includes any number that **can** be written as a fraction, and that includes some decimals. To know if a decimal number belongs in this set it either needs to be a terminating decimal (i.e. 0.0625 = 625/10000 = 1/16) or it needs to be a repeating decimal (i.e. 0.3333333…=1/3). The repetition string can be any length, but it does need to be one chunk of numbers being copied over and over again without change. This set is given the special symbol □ for "Quotient" (since they are all divisions of integers, and R is used below).

Irrational Numbers: This is the first of the sets we are looking at that does not subsume the previous set. Irrational numbers are any number you can think of that can be written as a decimal that does not terminate or repeat. Common elements of this set are square roots of numbers that aren't squares, like $\sqrt{2},\sqrt{5},\sqrt{\frac{7}{3}}$, etc., or any number that contains one of these. Also, certain numbers are special and come up in math all the time like $\pi \approx 3.1415928...$, $e \approx 2.7182818...,\varphi \approx 1.61803...$, together with any number that contains these numbers, like 1/e. Both π and e are in your calculator. The last, ϕ is the symbol for the Golden Ratio. Wikipedia can help you find out about it. This set is sometimes given the letter I as a symbol (and sometimes this is reserved for Imaginary numbers; we won't confuse them in this course, but make sure you know which set you are referring to when you use it).

Real numbers: Include all the numbers you can think of finding in the real world, including all the examples we've previously listed. This set has the special symbol □ .

(Pure) Imaginary Numbers: The word imaginary is sometimes used loosely for the set of Complex numbers so I want to distinguish it here with the "pure" label. Imaginary numbers are those numbers whose squares are negative numbers. No real number has such a property, that's why they are called "imaginary", but they are a perfectly valid system of numbers with their own rules, and are actually pretty handy in engineering. But, examples would include $\sqrt{-1}, \sqrt{-2}$, etc. The negative is sometimes pulled out and written as the lower case letter i.

Complex Numbers: This set of numbers includes both real numbers, imaginary numbers, and numbers using them in combination, such as 2+3i. Complex numbers are important for generalizing certain algebra properties, but not until you get to a little further. For now, just know that it's there. The symbol for it is □ .

We've dealt with all the important sets now, but I do want to say something about the set of undefined numbers, because we will encounter some of these. These are numbers that involve things like division by zero, or 0^0. These numbers sometimes result from the operations of numbers, but they are either undefined or indeterminate in value.

Practice: On the chart on the next page, for each of the sets, even the ones I didn't talk about here, come up with 3-5 numbers that belong in the set. Try not to use examples I've already used, but come up with some of your own. If the set contains subsets, your list should contain at least one element from each subset.

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