Mixture Problems in 2 & 3 variables

A common type of problem in multiple variables is a mixture problem. The problems involve one equation describing the mixture: the weights of the individual variables and how they contribute to the whole, together with the desired results. This equation is combined with one (or more) equations that describe a relationship between the variables. Typically, one of these equations that appears frequently is the sum equation, though any relationship between the components will do. Many types of problems fall under the category of mixture problems and can be constructed by similar means.

This handout will focus on setting up the problems. For solution techniques, see elsewhere.

The mixture equation generally takes one of two forms:

Quantity1*Weight1 + Quantity2*Weight2 = QuantityTotal*PriceTotal

OR

Quantity1*Weight1 + Quantity2*Weight2 = Total

And the sum equation looks like:

Quantity1 + Quantity2 = Total

Both of these are the case for two variables. In the case of three variables, we'll just have more than two similar terms on the left.

The "weights" may be prices, or percentages, or however the problem compares values. Other types of equations may be provided in the problem more explicitly as direct translation statements.

Let's consider some classic examples of mixture problems. We'll start with the 2-variable cases.

Example 1. Percent Solutions
A researcher working on an avian flu vaccine needs to mix a 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain 100 ml of a 30% sodium-iodine solution. How many milliliters of each solution should be used?

First, define your variables:
Let x be the quantity of 10% solution and y be the quantity of 60% solution.

The sum equation is the easiest to start with. We want 100 ml as the final quantity, which means that $\mathbf{x + y = 100}$.

Then we use the mixture equation:

Quantity1*Weight1 + Quantity2*Weight2 = QuantityTotal*PriceTotal

We use this form of it since we are given the total quantity and the weight (percent solution) of the final result separately.

X*0.10 + y*0.60 = (100)(0.30)
0.10x + 0.06y = 30

The system we must solve then contains the two equations: $\begin{cases} x + y = 100 \\ 0.10x + 0.60y = 30 \end{cases}$

We can then solve the system using whatever means we have available (substitution, elimination by addition, graphing, matrices… whatever we know).

Example 2. Dual Investment problems.
You have $50,000 to investment and your investment manager has suggested putting part in bonds and part in stocks to minimize risk. The stocks are expected to pay 15%, and the bonds 7% annually. You are expecting $6000 a year in additional income from the investment to help support your education. How much should be placed in each investment?

These problems are very similar to the percent solution problems except that the total is given as a single value, so we use the alternative mixture equation

Quantity1*Weight1 + Quantity2*Weight2 = Total

Let us call x the amount of money in bonds, and y the amount of money in stocks. Our mixture equation then becomes:

x*0.15 + y*0.07 = 6000
0.15x + 0.07y = 6000

The second equation is the sum equation: **x + y = 50,000**.

And so the system we must solve is $\begin{cases} x + y = 50{,}000 \\ 0.15x + 0.07y = 6000 \end{cases}$

Example 3. Theatre Tickets
A student theatre group has sold 3150 tickets for their play this season. Adult tickets were $6 and student tickets $4. They took in $15,600 in receipts. How many of each type of ticket did they sell?

Let x be the number of adult tickets and y be the number of student tickets sold. Since our total receipts are given as a single value, we use the same version of the mixture equation we used in the investment problem, and now the weights are not percents, but dollar figures for each ticket.

Quantity1*Weight1 + Quantity2*Weight2 = Total

x*6 + y*4 = 15,600
6x+4y = 15,600

The sum equation is also the second equation as before: **x + y = 3150**.

The system we must solve then is: $\begin{cases} 6x + 4y = 3150 \\ x + y = 3150 \end{cases}$

Example 4. Coffee Mixtures.
Besides theatre tickets, we can also mix things like fruit, coffee, nuts, vitamins and coins.

A store owner wants to make a medium-priced house coffee mixture from an inexpensive brand and a premium brand. The inexpensive brand costs $2.80 per pound and the premium brand costs $4.00 per pound. The owner wants 120 pounds of the house blend and wants the blend to sell for $3.20 per pound. If she wants to use twice as much of the inexpensive brand as the premium brand, how much of each of the original brands should she use?

Let x be the amount of the inexpensive brand, and let y be the amount of the premium brand. The mixture equation is:

Quantity1*Weight1 + Quantity2*Weight2 = QuantityTotal*PriceTotal

X*2.80 + y*4.00 = (120)(3.20)
2.8x + 4y = 384

Instead of the sum equation, we can use the fact that we want twice as much of the inexpensive brand as the premium one.

2y = x

We thus have the system: $\begin{cases} x = 2y \\ 2.8x + 4y = 384 \end{cases}$

Any of these problems can also be done with three variables, as long as we have three equations.

Example 5. A three-variable mixture problem.

A store owner wants to create a mixed nut blend from peanuts, cashews and macadamia nuts. The peanuts cost $2 per pound, the cashews $4 per pound, and the macadamia nuts are $7 per pound. The owner wants to sell the mixture for $5. If she wants to use twice as many peanuts as cashews, and to make 112 pounds of the mixture, how much of each nut (rounded to the nearest tenth of a pound) should be used in the mixture?

Let's start with establishing our variables: let x be the amount of peanuts, y be the amount of cashews, and z be the amount of macadamia nuts. The mixture equation is then:

Quantity1*Weight1 + Quantity2*Weight2 + Quantity3*Weight3 = QuantityTotal*PriceTotal

x*2 + y*4 + z*7 = (112)(5)
2x + 4y + 7z = 560

The sum equation show up in that she wants 112 pounds altogether: **x + y + z = 112**

And the third relationship we need is that we use twice as much peanuts as cashews: **x = 2y**.

Our three-variable system is then: $\begin{cases} x+y+z=112 \\ x=2y \\ 2x+4y+7z=560 \end{cases}$

Practice Problems:

1. A restaurant purchased 8 tablecloths and five napkins for $106. A week later, a tablecloth and six napkins cost $24. Find the cost of a single tablecloth and a single napkin. [Hint: use the mixture equation to set up both equations. Here, the quantity is known, but the weights are not, so they are the variables.]
2. A community center sells a total of 903 tickets for a basketball game. An adult ticket costs $3. A student ticket costs $2. The sponsors collect $1461 in ticket sales. Find the t total number of tickets sold of each type.
3. You invested $7000 in two accounts paying 6% and 8% annual interest. If the total interest earned was $520, how much was invested at each rate?
4. A wine company intends to blend a California wine with a 5% alcohol content and a French wine with a 9% alcohol content to obtain 200 gallons of 8% alcohol content. How many gallons of each type of wine (rounded to the nearest tenth of a gallon) must be used?
5. A jeweler needs to mix an alloy with 16% gold content with an alloy with a 28% gold content to obtain 32 ounces of an alloy with 25% gold content. How many ounces of each of the original alloys must be used?
6. Suppose the jeweler mixes the 16% alloy with pure gold to obtain the same 32 ounces of 25% gold content. How much of these must be used. [Hint: pure gold is 100% gold.]

7. A coin purse contains a mixture of 15 coins in dimes and quarters. The coins have a total value of $3.30. Determine the number of dimes and quarters in the purse.
8. A 40% dye solution is to be mixed with a 70% dye solution to obtain 120 L of a 50% solution. How many liters of each type of solution are needed?
9. Sally wishes to mix coffee worth $6 per pound with coffee worth $3 per pound to get 90 lbs. of a mixture worth $4 per pound. How many pounds each of the coffees are needed?
10. A nurse wishes to give a patient exactly 315 mg of a drug, but she has only pills coming in quantities of 45 mg per pill and 75 mg per pill. How many of each pill should she give the patient if she gives him 5 pills?