

Long Division of Polynomials.

Suppose I want to divide a trinomial by a polynomial, like this: $\frac{x^2 - 6x - 12}{x + 4}$. There are no factors of 12 that have a difference of 6, so the "reverse FOIL" method of factoring and cancelling factors won't work. We must do the long division. It's actually a lot like regular long division once you get the hang of it.

We can write the problem in a long division form like so: $x + 4 \overline{) x^2 - 6x - 12}$.

Step 1. Divide the leading term into the leading term.

This step is essentially asking us to simplify this expression

$$\frac{x^2}{x} = x \quad \text{Put the result over the same}$$

term above the division bar like so: $x + 4 \overline{) x^2 - 6x - 12}$

Step 2: Multiply this result by the divisor (in this case, that's x+4), and place it under the dividend (that's $x^2 - 6x - 12$)

$$x(x + 4) = x^2 + 4x \quad \text{so we get: } x + 4 \overline{) x^2 - 6x - 12}$$

$$x^2 + 4x$$

Step 3: Subtract.

$$x + 4 \overline{) x^2 - 6x - 12}$$

$$- (x^2 + 4x)$$

$$\longrightarrow$$

$$x + 4 \overline{) x^2 - 6x - 12}$$

$$\underline{-x^2 - 4x}$$

$$-10x$$

Step 4: Bring down the next term and repeat until you run out of terms.

We will need to do this one more time.

Step 1: Leading term into leading term: Simplify $\frac{-10x}{x} = -10 \longrightarrow x + 4 \overline{) x^2 - 6x - 12}$

Step 2: Multiply the result by the divisor. $-10(x+4) = -10x - 40$

$$\begin{array}{r}
 x-10 \\
 x+4 \overline{)x^2-6x-12} \\
 \underline{-x^2-4x} \\
 -10x-12 \\
 -10x-40
 \end{array}$$

Step 3: Subtract

$$\begin{array}{r}
 x-10 \\
 x+4 \overline{)x^2-6x-12} \\
 \underline{-x^2-4x} \\
 -10x-12 \\
 \underline{-(-10x-40)} \\
 28
 \end{array}
 \longrightarrow
 \begin{array}{r}
 x-10 \\
 x+4 \overline{)x^2-6x-12} \\
 \underline{-x^2-4x} \\
 -10x-12 \\
 10x+40 \\
 \hline
 28
 \end{array}$$

In this case, we ended up with a constant at the bottom, so there is a remainder of 28. We can't divide any more. In the nice cases we get a remainder of 0, but in those cases, we can factor (if we know how). In a case like the one above, this is the only way.