Loop Current Problems

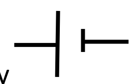
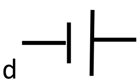
This handout will teach you one method of solving circuit diagram problems using loop currents. This is not the only method of solving these problems, but it is one common approach that uses linear algebra.

The solution is built on Kirchhoff's Law:

$$V = RI$$

When multiple batteries, currents and resistances are involved, each loop equation becomes

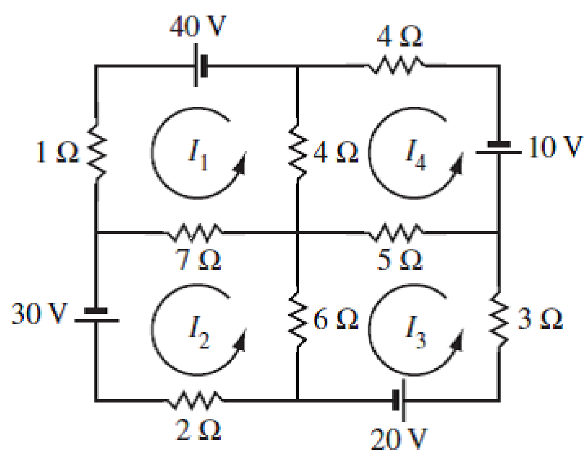
$$\sum_{i=1}^{n}\sum_{j=1}^{m} R_i I_j = \sum_{k=1}^{r} V_k$$

Signs are somewhat arbitrary and a matter of historical convention. For this handout, the convention for currents passing through a battery  is positive V, and  is negative. If doing these problems for a physics or electronic class, check for the local convention.

Because this is for a mathematics course, the default convention for the loop currents is for the loop to be oriented counterclockwise. This is a convention that may differ from course to course, so check the local convention.

The sign conventions will not change the magnitude of the currents obtained, but will change the sign.

Example 1. Solve the loop current problem shown in the diagram below.



If we begin with the current labeled I_1, choose a spot to begin (like the top right corner of the loop), to be sure that you don't miss any pieces of the equation. Going counterclockwise around the loop, we find first the 40 V battery. We are passing through it in the negative orientation, so -40 will go on the V-side of the equation.

Continuing around the loop, we find a 1Ω resistor. This will contribute $1I_1$ to the RI-side of the equation.

Next, we find a 7Ω resistor, which is shared by I_1 and I_2. Following the counterclockwise convention, I_1 is going left-to-right through this piece of wire, but I_2 is going right-to-left on its own counterclockwise loop. Therefore, we add $7I_1$ and $-7I_2$ to the RI-side of the equation.

Finally, we find a 4Ω resistor, which is shared by I_1 and I_4. As we did above, we add $4I_1$ and $-4I_4$ to the RI-side of the equation, since the two currents are going through that piece of the loop in opposite directions.

Altogether, then, we get

$$1I_1 + 7I_1 - 7I_2 + 4I_1 - 4I_4 = -40$$

Or when we collect like terms:

$$12I_1 - 7I_2 - 4I_4 = -40$$

Likewise, we can find the equations for the I_2 loop. There are $7 + 2 + 6 = 15\Omega$ resistors on I_2, and it interacts with I_3 on the 6Ω resistor, so $-6I_3$ and it interacts with I_1 on the 7Ω resistor, so $-7\mathrm{I}_1$. And the current passes through the 30V battery in the negative orientation. So, we get the equation

$$-7I_1 + 15I_2 - 6I_3 = -30$$

For the I_3 loop, we have $6 + 5 + 3 = 14\Omega$ resistors on I_3. Interacting with I_2 on the 6Ω resistor, and with I_4 on the 5Ω resistor. And it passes through the 20V battery in the negative orientation. So, we obtain the equation

$$-6I_2 + 14I_3 - 5I_4 = -20$$

Lastly, for the I_4 loop, there are $4 + 4 + 5 = 13\Omega$ resistors on I_4. It interacts with I_1on the 4Ω resistor, and with the I_3 current at the 5Ω resistor. It passes through the 10V battery in the positive orientation. This gives us the equation

$$-4I_1 - 5I_3 + 13I_4 = 10$$

It is possible to construct alternative loops (say around the whole thing, or around a pair of our currently labeled currents, but you will need 4 total equations, and 4 total loops to solve the system). If we put the equations together in a system, we can solve for each of the currents.

$$\begin{cases} 12I_1 - 7I_2 - 4I_4 = -40 \\ -7I_1 + 15I_2 - 6I_3 = -30 \\ -6I_2 + 14I_3 - 5I_4 = -20 \\ -4I_1 - 5I_3 + 13I_4 = 10 \end{cases}$$

Solve the system using row-operations.

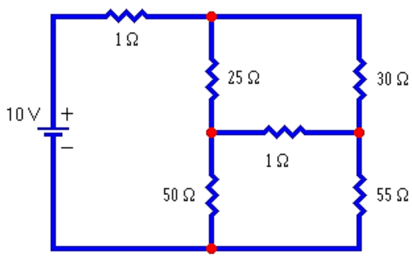
$$\left[\begin{array}{cccc|c} 12 & -7 & 0 & -4 & -40 \\ -7 & 15 & -6 & 0 & -30 \\ 0 & -6 & 14 & -5 & -20 \\ -4 & 0 & -5 & 13 & 10 \end{array}\right]$$

Solving, we obtain: $\vec{I} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -11.4342 \\ -10.5505 \\ -8.0357 \\ -5.8396 \end{bmatrix}$.

If we want then to determine the current in any particular section of the wire, we need only add the currents (using appropriate signs for their directions), in that particular section of the wire.

Practice problems.
Solve the loop current diagrams below using a system of equations. Round your answers to 4 decimal places (or 4 significant digits if using scientific notation).

1.



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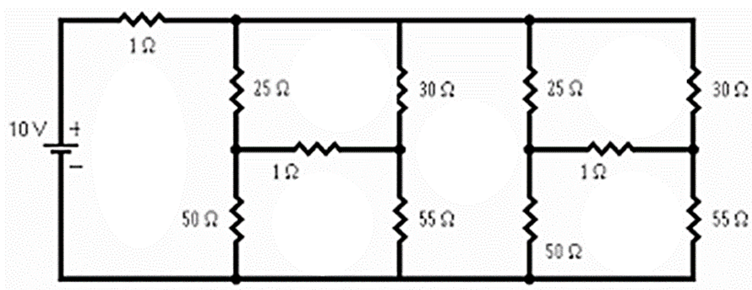

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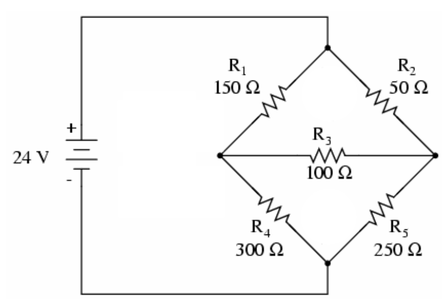

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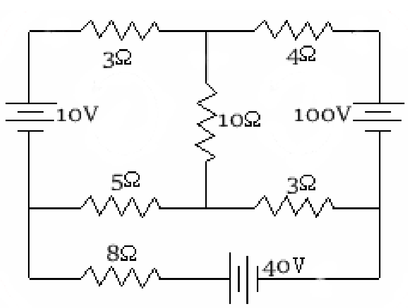

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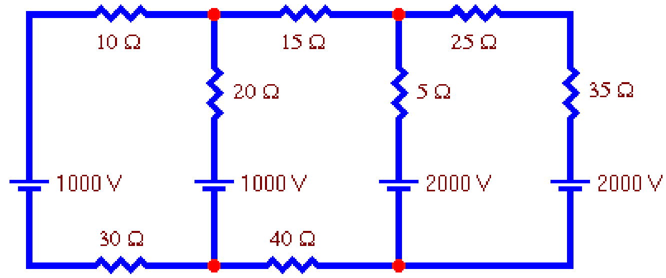

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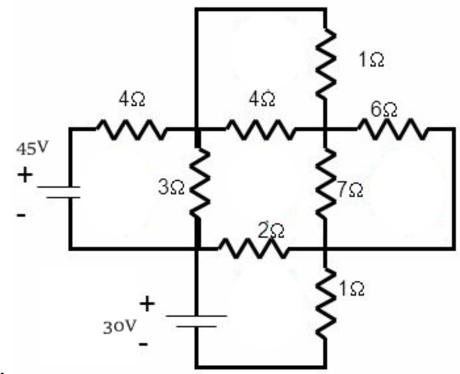

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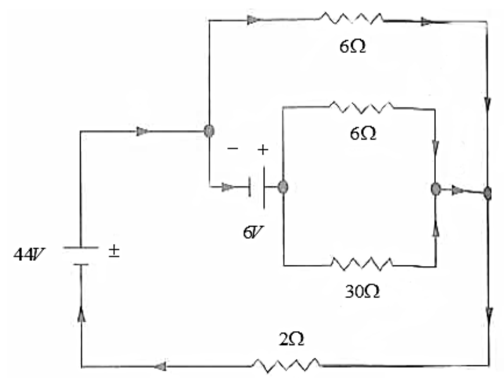

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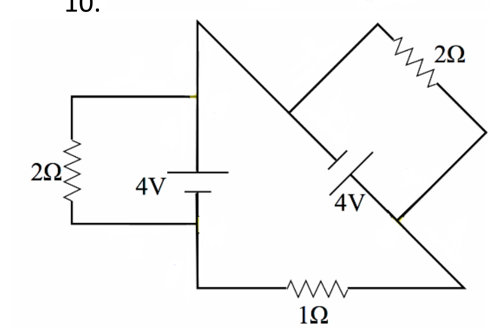

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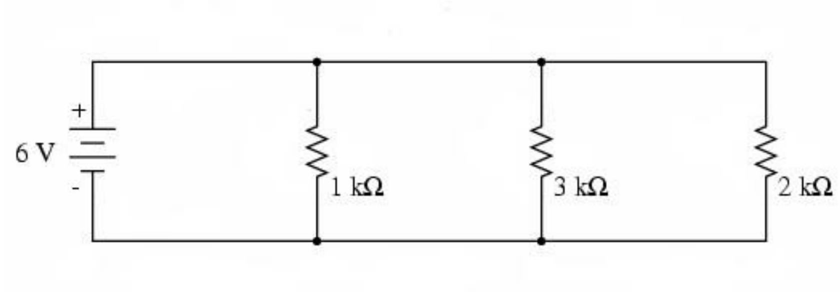
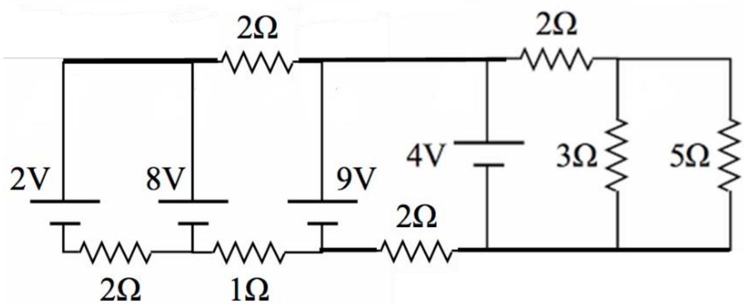
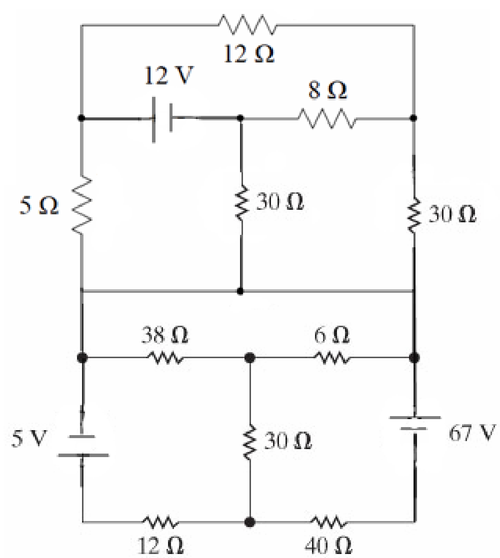

+
6 V
-
1 kΩ
3 kΩ
2 kΩ

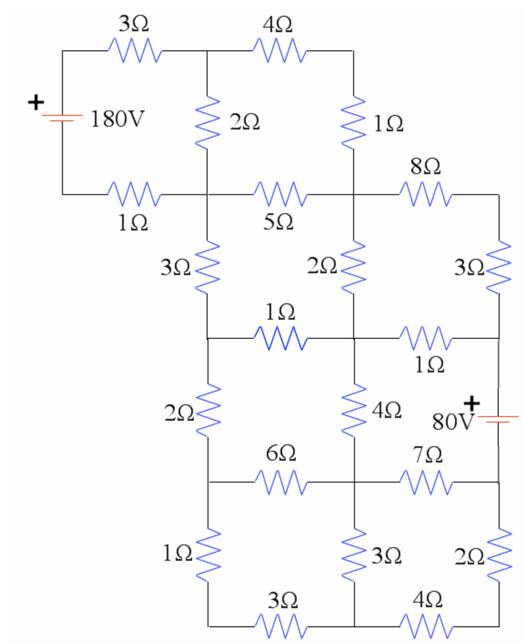
6.


2Ω
2Ω
2V
8V
9V
4V
3Ω
5Ω
2Ω
2Ω
1Ω

7.


12 Ω
12 V
8 Ω
5 Ω
30 Ω
30 Ω
38 Ω
6 Ω
5 V
30 Ω
67 V
12 Ω
40 Ω

12.


3Ω
4Ω
+
180V
2Ω
1Ω
8Ω
1Ω
5Ω
3Ω
2Ω
3Ω
1Ω
1Ω
2Ω
4Ω
+
80V
6Ω
7Ω
1Ω
3Ω
2Ω
3Ω
4Ω