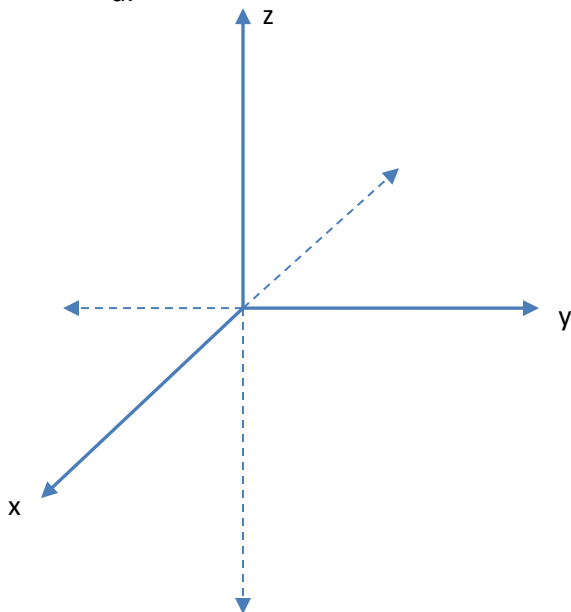


How to Plot points in 3D

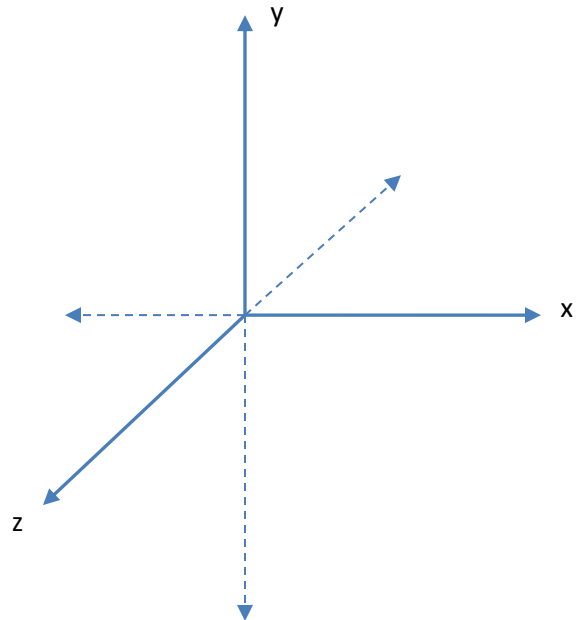


We will be using what is called the right-hand rule to graph in three dimensions. Hold out your right hand and point it in the direction of the positive x-axis. Your fingers curl in the direction of $+\theta$, and the positive y-axis. Your thumb points in the direction of the positive z-axis. See the image at left. On paper, this will either look like graph **a** or **b** below.

a.



b.

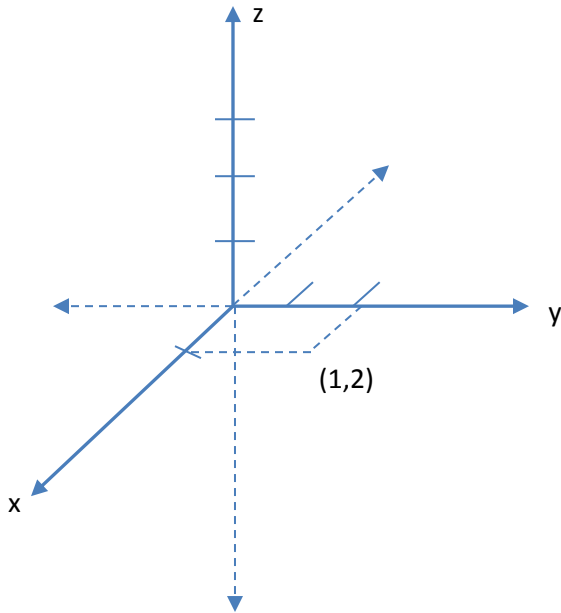


I don't care which of these methods you use, but you must label your graphs. I will not study them to figure out in which system your answer is most "right". My default orientation will be version **a**; however, computer generated graphs can be rotated freely, so you will need to look for the labels.

Problems, particularly with parameterized curves will ask us to draw the graph in 3D. We do this the same as for 2D, i.e. plot a series of points and connect the dots. A good graph of a helix will require 2 loops, which is a minimum of 9 points (4-5 per loop). But let's start with just one point.

Example 1. Let's try plotting the point $(1,2,3)$.

First, begin with your axes and label your tick marks.

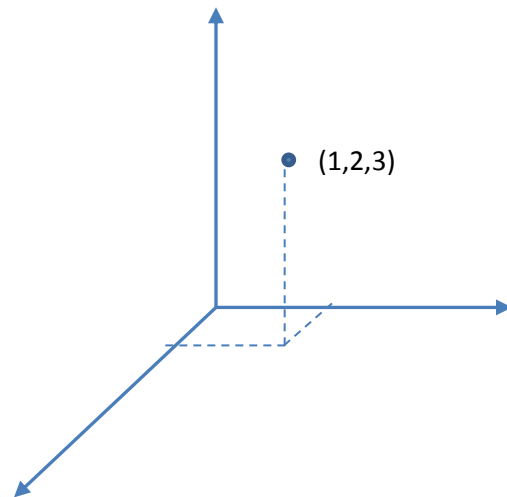


Count out along the graph one unit and draw a line parallel to the y-axis to represent the line $x=1$. Then count along the y-axis to represent the line $y=2$. Where they intersect, that is the point in the x-y plane $(1,2)$. Notice that the area marked out by these planes is a parallelogram and not a rectangle. This is because we are looking at the graph in perspective.

From the point $(1,2)$, we now want to count up 3 units in z, but we aren't counting up to where 3 is on the z-axis because we aren't on that plane anymore. We want to count 3 units up from the point in the x-y plane from the point $(1,2)$. This will account for the fact that we are looking at the graph in perspective. You can use your thumb and forefinger to "measure" the distance, or you can eyeball it.

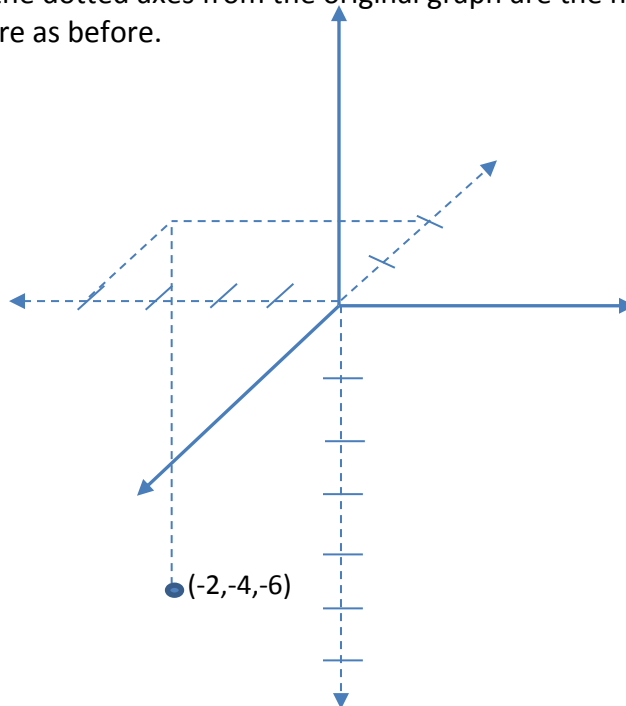
If you are plotting individual points, keep the dotted lines on the graph to maintain the 3D perspective appearance. If you are doing a large number of points, you can leave it off because connecting the points with a curve will provide the dimensional appearance.

Let's try another example.

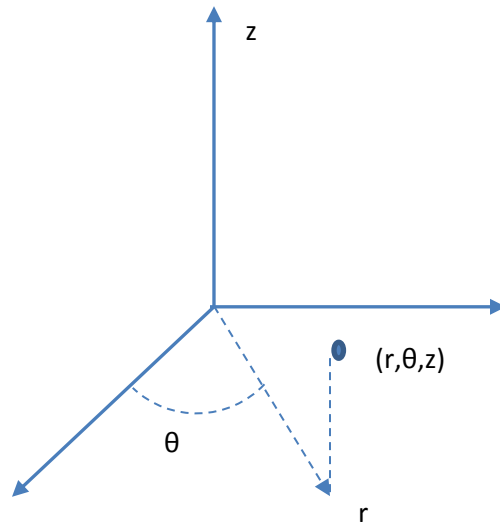


Example 2. Plot the point $(-2,-4,-6)$.

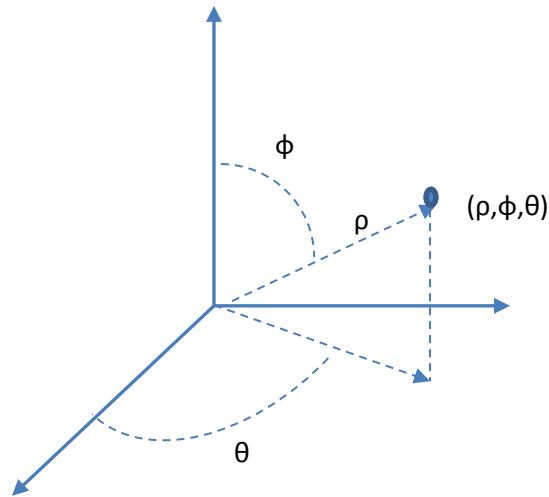
You'll notice that the dotted axes from the original graph are the negative directions. We do the same procedure as before.



In cylindrical coordinates, we still start with the same 3 axes. The angle between x and y is the angle θ , and z stays the same.



In spherical, the plane is like cylindrical coordinates, and the angle between the z axis and the plane is the angle ϕ , and ρ is the three-dimensional distance between the point and the origin. The negative z axis is $\pi/2$.



Practice Problems.

1. Plot the following points in rectangular coordinates. You may do so on the same graph if you wish.
 - a. $(0,0,3)$
 - b. $(2,1,6)$
 - c. $(-1,0,2)$
 - d. $(3,-4,1)$
 - e. $(2,2,0)$
2. Plot the following points in cylindrical coordinates. You may do so on the same graph if you wish.
 - f. $(2,\pi/3,4)$
 - g. $(3,\pi,-2)$
 - h. $(5,3\pi/4,1)$
3. Plot the following points in cylindrical coordinates. You may do so on the same graph if you wish.
 - i. $(2,\pi/2,\pi)$
 - j. $(4,\pi/6,\pi/2)$
 - k. $(5,4\pi/3,\pi/6)$
4. Graph the following parametric equations in 3D. For graphs that don't involve trigonometric functions, try $t=\{-3,-2,-1,0,1,2,3\}$. For points that contain trig functions you can try $t=\{-2\pi, -3\pi/2, -\pi, -\pi/2, 0, \pi/2, \pi, 3\pi/2, 2\pi\}$ or if the graph is less predictable, try $t=\{0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4, 2\pi\}$. Be sure to label each graph with an arrow on the curve in which t is increasing.
 - l.
 - m. (helix)
 - n.

o.