$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$

$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}.$

$\mathbf{v}(t) = \mathbf{r}'(t)$

$\|\mathbf{v}(t)\| = \frac{ds}{dt} = \|\mathbf{r}'(t)\|$

$\mathbf{a}(t) = \mathbf{r}''(t) = a_{\mathbf{T}}\mathbf{T}(t) + a_{\mathbf{N}}\mathbf{N}(t)$

$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$ and $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

$a_{\mathbf{T}} = \mathbf{a}\cdot\mathbf{T} = \frac{\mathbf{v}\cdot\mathbf{a}}{\|\mathbf{v}\|} = \frac{d^2s}{dt^2}$

$a_{\mathbf{N}} = \mathbf{a}\cdot\mathbf{N} = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|} = \sqrt{\|\mathbf{a}\|^2 - a_{\mathbf{T}}^2} = K\left(\frac{ds}{dt}\right)^2$

$K = \frac{|y''|}{[1 + (y')^2]^{3/2}}$

$K = \frac{|x'y'' - y'x''|}{[(x')^2 + (y')^2]^{3/2}}$

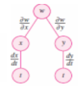
$K = \|\mathbf{T}'(s)\| = \|\mathbf{r}''(s)\|$

$K = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

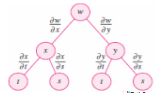
$K = \frac{\mathbf{a}(t)\cdot\mathbf{N}(t)}{\|\mathbf{v}(t)\|^2}$

CHAIN RULE:

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}.$$



CHAIN RULE: TWO



$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} \quad \text{and} \quad \frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}.$$

TANGENT PLANE Normal

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0.$$

AREA

$$A = \int_c^d \int_{h_1(y)}^{h_2(y)} dx\,dy.$$

SECOND PARTIALS TEST

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0.$$

$$d = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2.$$

$d > 0$: $f_{xx}(a, b) > 0$ relative minimum; $f_{xx}(a, b) < 0$ relative maximum

$d < 0$, saddle point.

inconclusive if $d = 0$.

VOLUME

$V = \int_R\int f(x, y)\,dA.$ $\quad Q = \int\int_Q\int dV.$

AVERAGE VALUE $\quad \frac{1}{A}\int_R\int f(x, y)\,dA$

PROJECTILE

$$\mathbf{r}(t) = (v_0\cos\theta)t\mathbf{i} + \left[h + (v_0\sin\theta)t - \frac{1}{2}gt^2\right]\mathbf{j}$$

JACOBIAN

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}.$$

GRADIENT $\nabla f(x, y) = f_x(x, y)\mathbf{i} + f_y(x, y)\mathbf{j}.$

DIRECTIONAL DERIVATIVE

$$D_{\mathbf{u}}f(x, y) = \nabla f(x, y)\cdot\mathbf{u}.$$

maximum $D_{\mathbf{u}}f(x, y)$ is $\|\nabla f(x, y)\|$.

minimum $D_{\mathbf{u}}f(x, y)$ is $-\|\nabla f(x, y)\|$.

$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$

$\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$

$\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$

:-D

CRITICAL POINT

$f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$

$f_x(x_0, y_0)$ or $f_y(x_0, y_0)$ does not exist.

SURFACE AREA

$$= \int_R\int \sqrt{1 + [f_x(x, y)]^2 + [f_y(x, y)]^2}\,dA.$$

MASS

$$m = \int_R\int \rho(x, y)\,dA.$$

$M_x = \int_R\int y\rho(x, y)\,dA$ and $M_y = \int_R\int x\rho(x, y)\,dA.$

$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right).$

LAGRANGE'S THEOREM

$$\nabla f(x_0, y_0) = \lambda\nabla g(x_0, y_0).$$

CONSERVATIVE PLANE

$$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}.$$

CONSERVATIVE SPACE

curl $\mathbf{F}(x, y, z) = \mathbf{0}.$

$$\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}, \quad \frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}, \quad \text{and} \quad \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}.$$

CURL

$\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is

curl $\mathbf{F}(x, y, z) = \nabla\times\mathbf{F}(x, y, z)$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}.$$

DIVERGENCE

$\mathbf{F}(x, y) = M\mathbf{i} + N\mathbf{j}$ is

$$\text{div}\,\mathbf{F}(x, y) = \nabla\cdot\mathbf{F}(x, y) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}. \quad \text{Plane}$$

$\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is

$$\text{div}\,\mathbf{F}(x, y, z) = \nabla\cdot\mathbf{F}(x, y, z) = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}. \quad \text{Space}$$

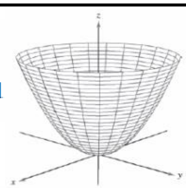
Power-Reducing

$\sin^2 u = \frac{1 - \cos 2u}{2}$ $\quad \cos^2 u = \frac{1 + \cos 2u}{2}$ $\quad \tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$

Ellipsoid

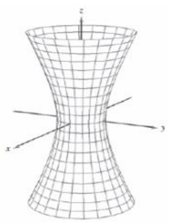
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Elliptic Paraboloid



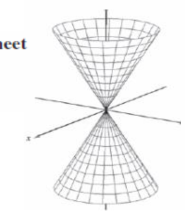
$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Hyperboloid of One Sheet



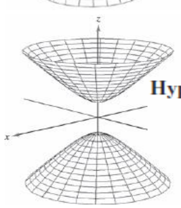
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Elliptic Cone



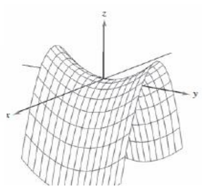
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

Hyperboloid of Two Sheets

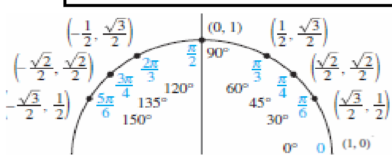


$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Hyperbolic Paraboloid



$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$



$$\int\int_Q\int f(x, y, z)\,dV = \int_{\theta_1}^{\theta_2}\int_{g_1(\theta)}^{g_2(\theta)}\int_{h_1(r\cos\theta,\, r\sin\theta)}^{h_2(r\cos\theta,\, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)r\,dz\,dr\,d\theta.$$

$$\int\int_Q\int f(x, y, z)\,dV = \int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2}\int_{\rho_1}^{\rho_2} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

Spherical to rectangular:

$x = \rho\sin\phi\cos\theta,$ $\quad y = \rho\sin\phi\sin\theta,$

$z = \rho\cos\phi$ $\quad r = \rho\sin\phi = \sqrt{x^2 + y^2}$

Rectangular to spherical:

$\phi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$

$\rho^2 = x^2 + y^2 + z^2$ $\quad \tan\theta = \frac{y}{x},$

Cylindrical to spherical ($r \geq 0$):

$\rho = \sqrt{r^2 + z^2},$ $\quad \theta = \theta,$ $\quad \phi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$

Spherical to cylindrical ($r \geq 0$):

$r^2 = \rho^2\sin^2\phi,$ $\quad \theta = \theta,$ $\quad z = \rho\cos\phi$

Sphere

Volume $= \frac{4}{3}\pi r^3$

Surface Area $= 4\pi r^2$

Sector of Circle

(θ in radians)

Area $= \frac{\theta r^2}{2}$

$s = r\theta$

Cylinder

Volume $= \pi r^2 h$

Lateral Surface Area $= 2\pi rh$

Circle

Area $= \pi r^2$

Circumference $= 2\pi r$

Ellipse

Area $= \pi ab$

Circumference $\approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$

Cone

(A = area of base)

Volume $= \frac{Ah}{3}$

LINE INTEGRAL

$$\int_C f(x, y)\, ds = \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(x_i, y_i)\, \Delta s_i \quad \text{Plane}$$

or

$$\int_C f(x, y, z)\, ds = \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(x_i, y_i, z_i)\, \Delta s_i \quad \text{Space}$$

DEFINITE INTEGRAL

$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, where $a \le t \le b$, then

$$\int_C f(x, y)\, ds = \int_a^b f(x(t), y(t))\sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt.$$

$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, where $a \le t \le b$, then

$$\int_C f(x, y, z)\, ds = \int_a^b f(x(t), y(t), z(t))\sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt.$$

$$\int_C 1\, ds = \int_a^b \|\mathbf{r}'(t)\|\, dt = \text{length of curve } C.$$

$$ds = \|\mathbf{r}'(t)\|\, dt = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\, dt.$$

VECTOR FIELD

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot \mathbf{T}\, ds = \int_a^b \mathbf{F}(x(t),\ y(t), z(t)) \cdot \mathbf{r}'(t)\, dt.$$

Differential Form

$$\begin{aligned}\int_C \mathbf{F} \cdot d\mathbf{r} &= \int_C \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}\, dt \\ &= \int_a^b (M\mathbf{i} + N\mathbf{j}) \cdot (x'(t)\mathbf{i} + y'(t)\mathbf{j})\, dt \\ &= \int_a^b \left(M\frac{dx}{dt} + N\frac{dy}{dt}\right) dt \\ &= \int_C (M\, dx + N\, dy)\end{aligned}$$

GREEN'S THEOREM

$$\int_C M\, dx + N\, dy = \int_R\!\!\int \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA.$$

AREA

$$A = \frac{1}{2}\int_C x\, dy - y\, dx.$$

FUNDAMENTAL THEOREM

open region

$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}, \quad a \le t \le b.$

$\mathbf{F}(x, y) = M\mathbf{i} + N\mathbf{j}$ is conservative

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r} = f(x(b), y(b)) - f(x(a), y(a))$$

PARAMETRIC SURFACE

$\mathbf{r}(u, v) = x(u, v)\mathbf{i} + y(u, v)\mathbf{j} + z(u, v)\mathbf{k}$ — Parametric surface

$x = x(u, v), \quad y = y(u, v), \quad$ and $\quad z = z(u, v)$ — Parametric equations

NORMAL VECTOR

$\mathbf{r}(u, v) = x(u, v)\mathbf{i} + y(u, v)\mathbf{j} + z(u, v)\mathbf{k}$

$(x_0, y_0, z_0) = (x(u_0, v_0), y(u_0, v_0), z(u_0, v_0))$

$$\mathbf{N} = \mathbf{r}_u(u_0, v_0) \times \mathbf{r}_v(u_0, v_0) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v}\end{vmatrix}.$$

AREA

$\mathbf{r}(u, v) = x(u, v)\mathbf{i} + y(u, v)\mathbf{j} + z(u, v)\mathbf{k}$

$$\text{Surface area} = \int_S\!\!\int dS = \int_D\!\!\int \|\mathbf{r}_u \times \mathbf{r}_v\|\, dA$$

where $\mathbf{r}_u = \frac{\partial x}{\partial u}\mathbf{i} + \frac{\partial y}{\partial u}\mathbf{j} + \frac{\partial z}{\partial u}\mathbf{k}$ and $\mathbf{r}_v = \frac{\partial x}{\partial v}\mathbf{i} + \frac{\partial y}{\partial v}\mathbf{j} + \frac{\partial z}{\partial v}\mathbf{k}.$