

# Partial Fractions

$$i. \frac{3x+1}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} = \frac{1}{x-2} + \frac{2}{x+5}$$

$$A(x+5) + B(x-2) = 3x+1$$

if  $x=2$

$$A(7) = 7 \quad A=1$$

if  $x=-5$

$$B(-7) = -14 \quad B=2$$

$$ii. \frac{x^2+11x}{x^2+5x+6} \quad \begin{array}{r} x^2+5x+6 \overline{) x^2+11x} \\ -x^2-5x-6 \\ \hline 6x-6 \end{array} \quad x^2+5x+6 = (x+2)(x+3)$$

$$1 + \frac{6x-6}{(x+2)(x+3)}$$

$$A(x+3) + B(x+2) = 6x-6$$

$$1 + \frac{A}{x+2} + \frac{B}{x+3}$$

if  $x=-3$

$$B(-1) = -24 \quad B=24$$

if  $x=-2$

$$A(1) = -18 \quad A=-18$$

$$1 - \frac{18}{x+2} + \frac{24}{x+3}$$

$$iii. \frac{1}{x^3-1} = \frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$A(x^2+x+1) + (Bx+C)(x-1) = 1$$

if  $x=1$

$$A(3) = 1 \quad A = \frac{1}{3}$$

if  $x=0$

$$A(1) + C(-1) = 1$$

$$\Rightarrow \frac{1}{3} + C = 1 \quad C = \frac{2}{3}$$

$$-C = \frac{2}{3} \quad C = -\frac{2}{3}$$

if  $x=-1$

$$A(1) + (-B+C)(-2) = 1$$

$$\frac{1}{3} + 2B + \frac{4}{3} = 1$$

$$\Rightarrow \frac{5}{3} + 2B = 1 \quad 2B = -\frac{2}{3}$$

$$2B = -\frac{2}{3}$$

$$B = -\frac{1}{3}$$

$$\frac{1}{3} + \frac{-\frac{1}{3}x - \frac{2}{3}}{x^2+x+1}$$

$$iv. \frac{x^2-6}{(x+1)(x+4)(x-3)} = \frac{A}{x+1} + \frac{B}{x+4} + \frac{C}{x-3} = \frac{5/12}{x+1} + \frac{10/21}{x+4} + \frac{3/28}{x-3}$$

$$A(x+4)(x-3) + B(x+1)(x-3) + C(x+1)(x+4) = x^2-6$$

if  $x = -1$

$$A(3)(-4) = -5 \quad A = \frac{5}{12}$$

if  $x = -4$

$$B(-3)(-7) = 10 \quad B = \frac{10}{21}$$

if  $x = 3$

$$C(4)(7) = 3 \quad C = \frac{3}{28}$$

$$v. \frac{2x+3}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{2}{x-1} + \frac{5}{(x-1)^2}$$

$$A(x-1) + B = 2x+3$$

$$A(x-1) + \frac{5}{-5} = \frac{2x+3}{-5}$$

$x=1$

$$B = 5$$

$$A(x-1) = 2x-2$$

$$A = 2$$

$$vi. \frac{x^2+2x+4}{(x-3)^2(x+1)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+1} = \frac{13/16}{x-3} + \frac{19/4}{(x-3)^2} + \frac{3/16}{x+1}$$

$$A(x-3)(x+1) + B(x+1) + C(x-3)^2 = x^2+2x+4$$

if  $x = 3$

$$B(4) = 9+6+4 = 19$$

$$B = \frac{19}{4}$$

if  $x = -1$

$$C(-4)^2 = 1-2+4 = 3$$

$$C = \frac{3}{16}$$

if  $x = 0$

$$A(-3)(1) + B + C(-3)^2 = 4$$

$$-3A + \frac{19}{4} + \frac{27}{16} = 4$$

$$-3A = -\frac{39}{16} \quad A = \frac{13}{16}$$

$$vii. \frac{x^3+4x+1}{(x-2)^3(x+7)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} + \frac{D}{x+7}$$

$$(x-2)^3(x+7) = (x-2)^3(x+7) = x^3+4x+1$$

$$\text{viii. } \frac{4x+13}{(x^2+4)(x-3)^2} = \frac{Ax+B}{x^2+4} + \frac{C}{x-3} + \frac{D}{(x-3)^2}$$

$$(Ax+B)(x-3)^2 + C(x^2+4)(x-3) + D(x^2+4) = 4x+13$$

if  $x=3$

$$D(13) = 25 \quad D = 25/13$$

if  $x=0$

$$B(-3)^2 + C(4)(-3) + D(4) = 13$$

$$9B - 12C + 4D = 13$$

if  $x=1$

$$(A+B)(-2)^2 + C(5)(-2) + D(5) = 17$$

$$9B - 12C + \frac{100}{13} = 13$$

$$9B - 12C = 69/13$$

$$4A + 4B - 10C + 5D = 17$$

$$3B - 4C = 23/13$$

$$4A + 4B - 10C + \frac{125}{13} = 17$$

$$4A + 4B - 10C = 96/13$$

if  $x=-1$

$$(-A+B)(-4)^2 + C(5)(-4) + D(5) = 9$$

$$2A + 2B - 5C = 48/13$$

$$-16A + 16B - 20C + \frac{125}{13} = 9$$

$$-16A + 16B - 20C = -8/13$$

$$-4A + 4B - 5C = -2/13$$

$$3B - 4C = 23/13$$

$$2A + 2B - 5C = 48/13$$

$$\rightarrow 6A - 2B = 50/13$$

$$3A - B = \frac{25}{13}$$

$$4A - 4B + 5C = 2/13$$

$$3A = B + \frac{25}{13}$$

$$4A + 4B - 10C = 48/13$$

$$\rightarrow 8B - 15C = 46/13$$

$$-4A + 4B - 5C = -2/13$$

$$24B - 32C = \frac{184}{13}$$

$$\rightarrow 13C = \frac{46}{13}$$

$$C = \frac{46}{169}$$

$$-24B + 45C = \frac{-138}{13}$$

$$3A = \frac{161}{169} + \frac{25}{13}$$

$$3A = \frac{486}{169}$$

$$A = \frac{162}{169}$$

$$3B = \frac{23}{13} + 4\left(\frac{46}{169}\right) = \frac{483}{169}$$

$$B = \frac{161}{169}$$

$$\frac{\frac{162}{169}x + \frac{161}{169}}{x^2+4} + \frac{\frac{46}{169}}{x-3} + \frac{\frac{25}{13}}{(x-3)^2}$$

$$\text{ix. } \frac{x^2+3x-10}{(x^2+2)^2(x-9)} = \frac{Ax+B}{(x^2+2)} + \frac{Cx+D}{(x^2+2)^2} + \frac{E}{(x-9)}$$

$$\text{x. } \frac{x^3+5x^2-8x-24}{(x^2+1)^2(x-3)^3(x+4)(x-1)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{x-3} + \frac{F}{(x-3)^2} + \frac{G}{(x-3)^3} \\ + \frac{H}{(x+4)} + \frac{I}{(x-1)}$$

$$\text{xi. } \int \frac{5x-12}{(x^2-4)} dx = \int \frac{5x-12}{(x-2)(x+2)} dx = \int \frac{A}{x-2} + \frac{B}{(x+2)} dx$$

$$A(x+2) + B(x-2) = 5x-12$$

if  $x=2$

$$4A = -2 \quad A = -\frac{1}{2}$$

if  $x=-2$

$$-4B = -22 \quad B = \frac{11}{2}$$

$$\int \frac{-\frac{1}{2}}{x-2} + \frac{\frac{11}{2}}{x+2} dx = -\frac{1}{2} \ln|x-2| + \frac{11}{2} \ln|x+2| + C$$

$$\text{xii. } \int \frac{2x-3}{(x^2+16)(x-2)} dx = \int \frac{Ax+B}{x^2+16} + \frac{C}{x-2} dx$$

$$(Ax+B)(x-2) + C(x^2+16) = 2x-3$$

if  $x=2$

$$C(20) = 3 \quad C = \frac{3}{20}$$

if  $x=0$

$$B(-2) + C(16) = -3 \quad -2B = -3 - \frac{48}{20} \quad B = \frac{27}{10}$$

if  $x=1$

$$(A + \frac{27}{10})(-1) + \frac{3}{20}(17) = -1 \quad A + \frac{27}{10} = \frac{71}{20} \quad A = \frac{-25}{4}$$

$$-\frac{25}{4} \int \frac{x}{x^2+16} dx + \frac{27}{10} \int \frac{1}{x^2+16} dx + \frac{3}{20} \int \frac{1}{x-2} dx$$

$$-\frac{25}{8} \ln|x^2+16| + \frac{27}{40} \arctan\left(\frac{x}{4}\right) + \frac{3}{20} \ln|x-2| + C$$

$$\text{xiii. } \int \frac{10x^2 - x - 17}{x^2(x+3)(x-1)} dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3} + \frac{D}{x-1} dx$$

$$A(x+3)(x-1) + B(x+3)(x-1) + Cx^2(x-1) + Dx^2(x+3) = 10x^2 - x - 17$$

if  $x=0$

$$B(3)(-1) = -17 \quad B = \frac{17}{3}$$

if  $x=1$

$$D(+1)^2(4) = 10 - 1 - 17 \quad 4D = -8 \quad D = -2$$

if  $x=-3$

$$C(9)(2) = 90 - 3 - 17 \quad 18C = 70 \quad C = \frac{35}{9}$$

if  $x=-1$

$$A(-1)(2)(-2) + B(2)(-2) + C(1)(-2) + D(+1)(2) = 10 + 1 - 17$$

$$4A - 4B - 2C + 2D = -6$$

$$4A - \frac{68}{3} - \frac{70}{9} - 4 = -6 \quad 4A = \frac{256}{9} \quad A = \frac{64}{9}$$

$$\frac{64}{9} \int \frac{1}{x} dx + \frac{17}{3} \int \frac{1}{x^2} dx + \frac{35}{9} \int \frac{1}{x+3} dx + -2 \int \frac{1}{x-1} dx$$

$$\frac{64}{9} \ln|x| - \frac{17}{3x} + \frac{35}{9} \ln|x+3| - 2 \ln|x-1| + C$$