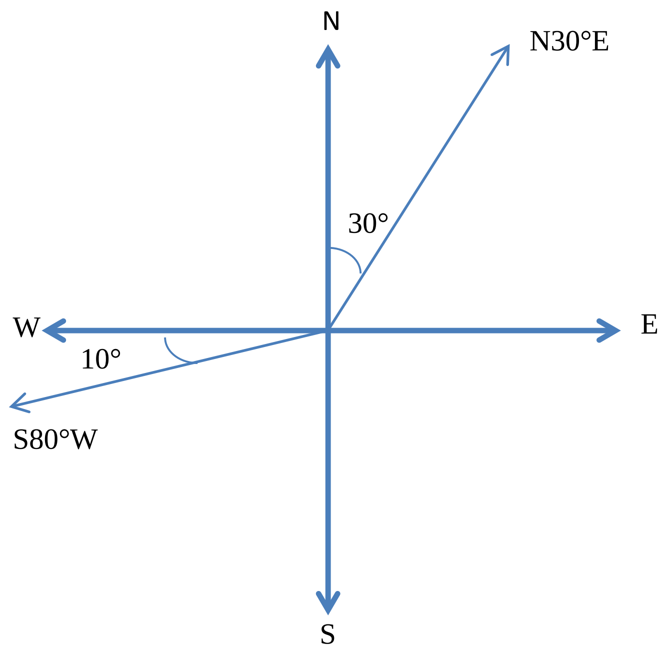
Bearings Problems

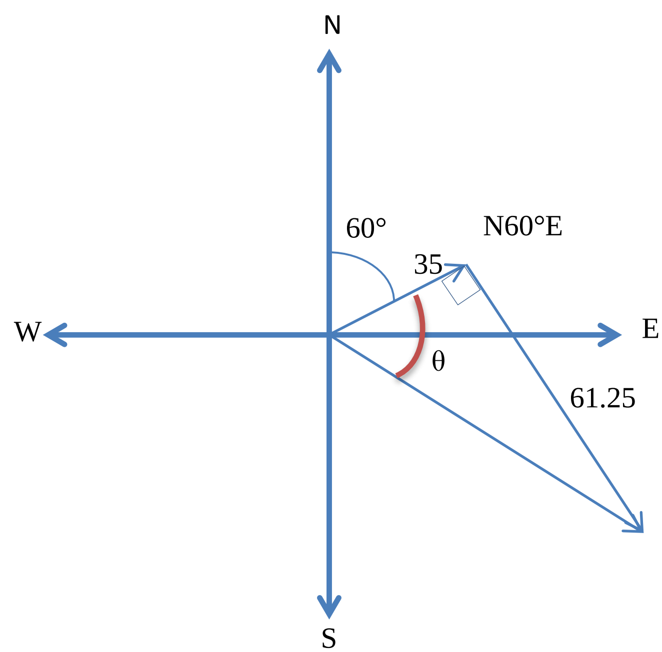
One application of trigonometry has to do with dealing with directions on the compass. One way of dealing with compass directions is to describe true north as 0°, and then increasing clockwise around to 359° before arriving at north again. These types of problems are similar enough to traditional angles in standard form (with angles relative to the positive y-axis, and switching sine and cosine relations) that they are relatively easy to translate.

However, another way of doing compass directions is to divide the directions into 4 quadrants of 90° each. In each quadrant, the angles are measured from the North-South axis, and either East or West is noted depending on whether the angle is measured to the right or left of the main axis. For instance, N30°E measures a 30° angle toward the east from true north. Directions are never measured from the east or west axes. When you have that kind of information, you must subtract the angle from 90° to calculate the direction from the other axis, as shown to the right. i.e. 10° south of due west is given as S80°W, or 80° to the west of due south. Furthermore, as we will see in the examples below, if the angle should flip to a different quadrant, we must recalculate the angle from the other side of the north-south axis.



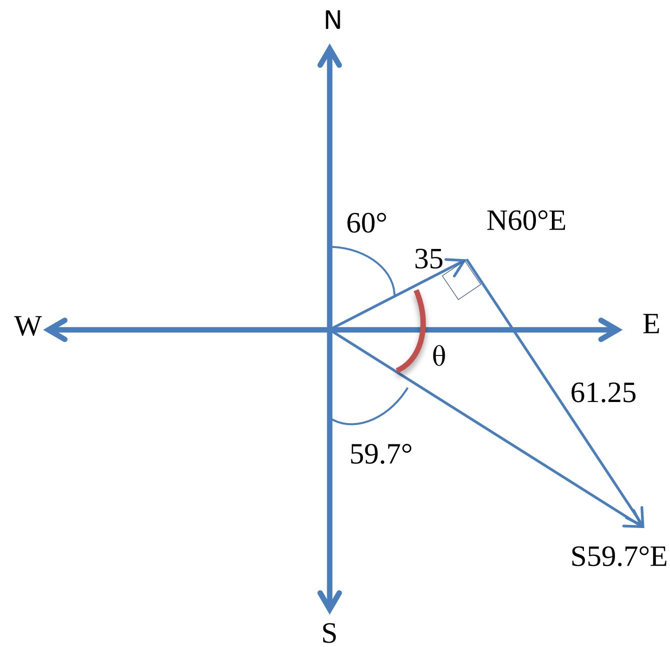
This handout will concern itself with problems using this 4-quadrant method of doing direction angles.

Example 1.
Suppose that a plane starts out travelling N60°E at 35 mph. After an hour, the plan takes a 90° turn toward the south and travels for another hour and 45 minutes. If the radio tower wishes to contact the plane at that point, in which direction should they transmit the signal?



It helps to begin drawing a diagram of the situation.

During the first hour, the plane traveled 35 miles. During the second leg, it traveled $1.75 \times 35 = 61.25$ miles. (Note that we converted 1 hour and 45 minutes to 1 and ¾ hours or 1.75 hours to find the distance traveled.) To find the new direction, we need to find the angle marked in red.

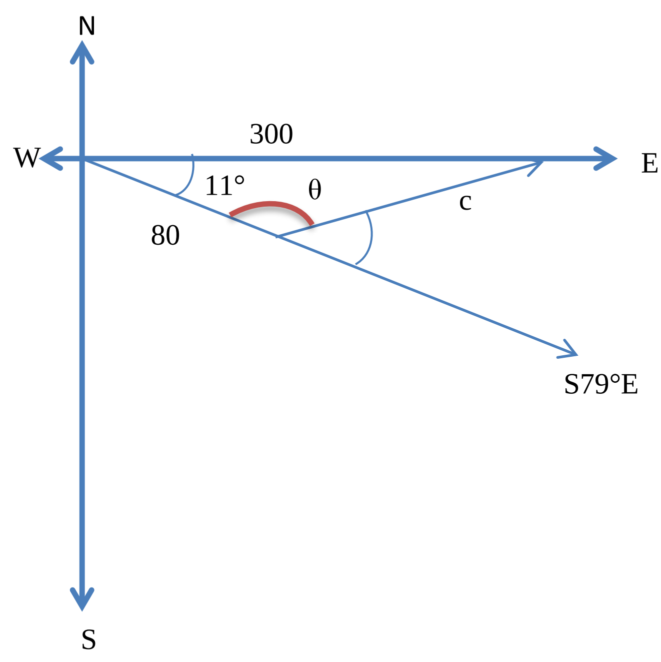


Since we have the two sides of the triangle that corresponds to the side opposite the angle (61.25) and the side adjacent the angle (35), we can calculate the tangent of the angle θ to be $\tan\theta = \frac{61.25}{35}$, or $\theta \approx 60.3°$.

Since the bearing was originally N60°E, there are 30° left to due east, we subtract that off the angle we found to see how far into the SE quadrant we are in. 60.3°-30°=30.3°. Since we also can't measure our direction from the eastern direction and must measure it from the southern axis, we subtract this angle from 90°: 90°-30.3°=59.7° east of south. So our final direction is S59.7°E.

Example 2.

Suppose that a boat leaves an island travelling for another 300 nautical miles due east of where he launches from. However, the ocean current pushes him off course by an angle of 11°. If he's travelling at 40 knots, and doesn't discover that he's being carried off course for 2 hours, in what direction should he travel in order to reach his destination?



If we want to calculate his new direction, we need to find the other angles in the triangle, particularly the large one marked in red. To find this angle, we first have to find the length of the missing side (how far away he is from his destination when he changes direction). For this, we use the law of cosines.

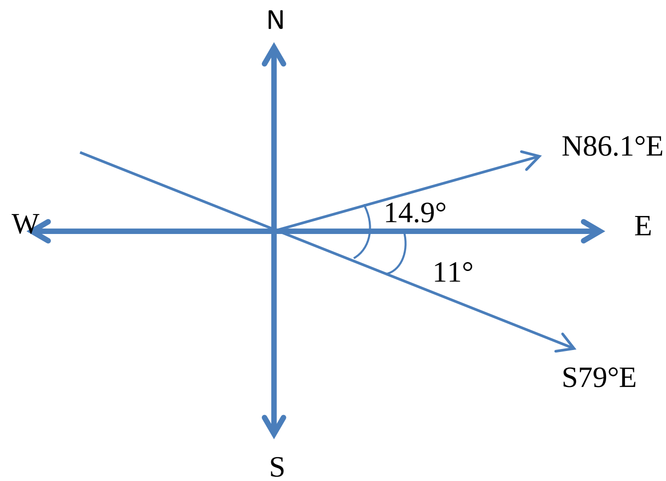
$$c^2 = a^2 + b^2 - 2ab\,cos\theta$$
$$c^2 = 80^2 + 300^2 - 2(80)(300)\cos(11°)$$
$$c = 222$$

Then we use the law of sines to find the angle we need (note that it's obtuse and so we will need a second quadrant angle).

$$\frac{\sin\theta}{300} = \frac{\sin 11°}{222}$$

$$\theta \approx 165.1°$$

This means that the angle the boat will have to turn through is 14.9°. To see how this gives us the bearing information, we can move the coordinate axis to the point where the boat changes direction.



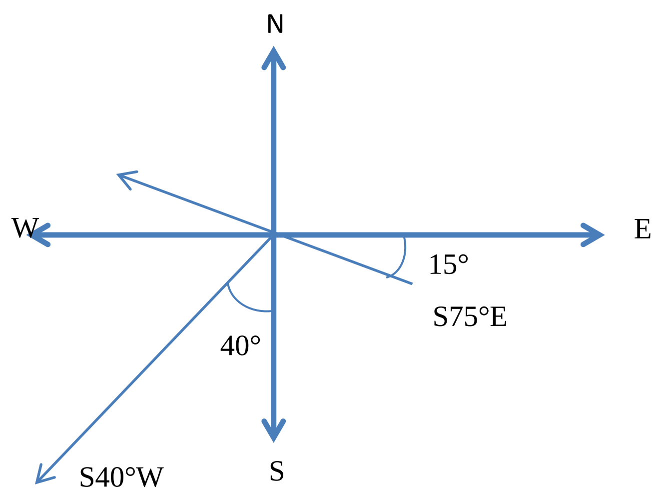
Since we were pointing 11° south of due east and the angle we are turning through is larger than this, we will need to find how much is left to get the angle we need in the NE quadrant. 14.9°-11°=3.9°.

Once the boat makes the turn it will be pointing into an angle 3.9° north of due east. To calculate the direction angle from due north, we need to subtract this angle from 90°. 90°-3.9°=86.1°.

So our final direction angle is N86.1°E.

We can also encounter bearings problems with vectors.

Example 3.
Suppose that a plane flies at a bearing of S40°W at a speed of 100 km/h into a crosswind coming *from* S75°E at 15 km/h. Find the resulting speed and direction of the plane.



To add these velocities, we are going to need to convert them to standard angles.

The plane's direction is given in the direction it is flying into, so we just need to figure the angle from due east (the positive x-axis). Since it's in the SW quadrant, we add the direction angle (40°) to 90° for the negative angle, or -130°. (It's negative, remember, because we are measuring it clockwise instead of counterclockwise.)

The wind direction is normally given by the direction it's coming **from**. To add the vectors, we will need the direction it's heading into. So instead of S75°E, it's pointing in N75°W. Since this is a western angle, we likewise add 90° to measure from due east, or 90°+75°=165°. This is a positive angle since we are measuring it counterclockwise.

We must convert the vectors to be added into rectangular form.

Plane: $100(\cos(-130°)\hat{\boldsymbol{\imath}} + \sin(-130°)\,\hat{\boldsymbol{\jmath}}) \approx -64.28\hat{\imath} - 76.60\hat{\boldsymbol{\jmath}}$
Wind: $15(\cos(165°)\,\hat{\boldsymbol{\imath}} + \sin(165°)\hat{\boldsymbol{\jmath}}) \approx -14.49\hat{\imath} + 3.88\hat{\boldsymbol{\jmath}}$

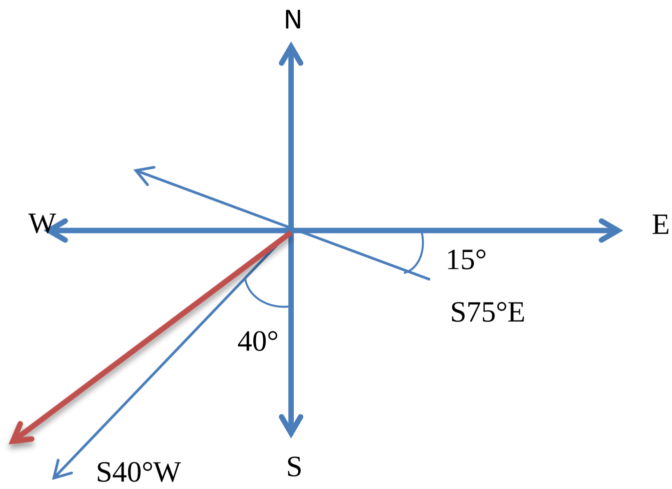
Plane+Wind: $-78.77\hat{\boldsymbol{\imath}} - 72.72\hat{\boldsymbol{\jmath}}$

To determine the magnitude of the vector, we find: $\sqrt{78.77^2 + 72.72^2} \approx 107.2$ km/h.

Since both components in rectangular form are negative, we will need to find an angle in the third quadrant. Here then: $\tan\theta = \frac{-72.72}{-78.77}$ or $\theta + 180° = 222.7°$. $(tan^{-1}\left(\frac{72.72}{78.77}\right) \approx 42.7°)$

To calculate the angle from due south, we subtract the 222.7° angle from 270°. 270°-222.7°=47.3°

The resulting vector is shown on the graph in red. The true bearing of the plane in the crosswind is S47.3°W.



Practice Problems.

1. Station Able is located 150 miles due south of Station Baker. A ship sends an SOS. Station Able receives the call and indicates it is at a bearing of N50°E. Station Baker reports the same SOS from the direction of S75°E. How far is the ship from each station? If rescue ships from both stations travel at the same speed, from which ship should the rescue boat be sent?
2. Alan must fly to St. Louis from Oklahoma City. One option is to fly direct, a distance of 461.1 miles, bearing N57.7°E. Another option flies first to Kansas City (bearing N29.6°E) and then to St. Louis (bearing N79.4°E). Find the distance along each route.
3. An airplane flies due north from Fort Myers to Sarasota, a distance of 150 miles, and then turns through an angle of 50° to go to Orlando another 100 miles further on. What is the bearing and distance the plane would have to fly in if it traveled directly to Orlando?
4. A cruise ship maintains an average speed of 20 knots going from San Juan, Puerto Rico to Barbados, West Indies, a distance of 600 miles at a bearing of S85°E. Suppose the ship is pushed off course through an angle of 24°, but does not discover it until travelling for 14 hours. What bearing must the ship change its direction to end up in Barbados as planned.
5. A 747 leaves Midway Airport from Runway 4 Right whose bearing is N40°E. After flying for half a mile, the pilot requests permission to turn 90° toward the SE. After going a mile in this direction, what is the bearing of the plane from the airport tower?
6. A ship leaves the port of Miami with a bearing of S80°E and a speed of 30 knots. After an hour, the ship turns through an angle of 90° toward the south, and changes speed to 20 knots and continues on that path for 4 hours. What is the bearing of the ship to the port at that time?
7. An airplane has an airspeed of 600 km/h bearing S35°W. The wind is 40 km/h from a direction of S45°E. What is the groundspeed and direction of the plane?
8. An airplane has an airspeed of 50 mph bearing N15°W. The wind is coming at 30 mph from a direction of S20°E. What is the groundspeed and direction of the plane?
9. Suppose a plane wishes to travel 100 miles in a direction of S55°E. The wind is coming from a direction of 25 mph from the direction N65°E. At what speed and in what direction must the plan travel to arrive at the destination in 2 hours?