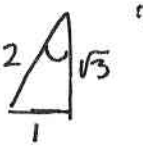
Solving Trig Equations Key Math 1149

i. $2\sin\theta + 1 = 0 \Rightarrow 2\sin\theta = -1 \Rightarrow \sin\theta = -\frac{1}{2}$



$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

ii. $4\sec\theta + 6 = -2 \Rightarrow 4\sec\theta = -8 \Rightarrow \sec\theta = -2 \Rightarrow \cos\theta = -\frac{1}{2}$ $\pi/3$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

iii. $\tan\theta + 1 = 0 \Rightarrow \tan\theta = -1$ $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

iv. $\sin(3\theta) = -1$ $3\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

v. $\cot\left(\frac{2\theta}{3}\right) = -\sqrt{3}$ $\frac{2\theta}{3} = \frac{5\pi}{6}, \frac{11\pi}{6}$ $\pi/6$

$\theta = \frac{5\pi}{6} \cdot \frac{3}{2} = \frac{5\pi}{4}, \quad \frac{11\pi}{6} \cdot \frac{3}{2} = \frac{11\pi}{4} > 2\pi$

vi. $\cos\left(2\theta - \frac{\pi}{2}\right) = -1$

$2\theta - \frac{\pi}{2} = \pi, 3\pi, 5\pi$

$2\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2} \Rightarrow \frac{3\pi}{4}, \frac{7\pi}{4} = \theta$

vii. $\sin\left(3\theta + \frac{\pi}{18}\right) = 1$

$3\theta + \frac{\pi}{18} = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

$3\theta = \frac{4\pi}{9}, \frac{22\pi}{9}, \frac{40\pi}{9}$

$\theta = \frac{4\pi}{27}, \frac{22\pi}{27}, \frac{40\pi}{27}$

viii. $\tan(2\theta) = -1$

$2\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

$\theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

ix. $\tan\theta = 5$ $\theta = \tan^{-1}(5) \approx 1.3734, 4.5150$

x. $\csc\theta = -3 \Rightarrow \sin\theta = -\frac{1}{3} \Rightarrow \theta = \sin^{-1}\left(-\frac{1}{3}\right) \approx 3.4814, 5.9433$

xi. $\sin 2\theta - 1 = 0 \Rightarrow \sin^2\theta = 1 \Rightarrow \sin\theta = \pm 1 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$

xii. $\cot\theta + 1 = 0 \Rightarrow \cot\theta = -1 \Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

$2\csc\theta - 1 = 0 \Rightarrow 2\csc\theta = 1 \Rightarrow \csc\theta = \frac{1}{2}$ no solution

Xiii. $4\cos^2\theta - 3 = 0 \Rightarrow 4\cos^2\theta = 3 \Rightarrow \cos^2\theta = \frac{3}{4} \Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$ (2)

$\pi/6$

$\theta = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

Xiv. $\tan^2\theta = 3 \Rightarrow \tan\theta = \pm\sqrt{3}$ $\theta = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$ $\pi/3$

Xv. $\sin^2\theta = 6(\cos\theta + 1) \Rightarrow 1 - \cos^2\theta = 6\cos\theta + 6 \Rightarrow 0 = \cos^2\theta + 6\cos\theta + 5$

$(\cos\theta + 5)(\cos\theta + 1) = 0$

$\cos\theta + 5 = 0 \Rightarrow \cos\theta = -5$ no solution

$\cos\theta + 1 = 0 \Rightarrow \cos\theta = -1 \Rightarrow \theta = \pi$

Xvi. $\cos 2\theta = 2 - 2\sin^2\theta \Rightarrow 1 - 2\sin^2\theta = 2 - 2\sin^2\theta \Rightarrow 1 = 2$

no solution

Xvii. $\tan\theta = \sin\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \sin\theta \Rightarrow \sin\theta = \cos\theta\sin\theta \Rightarrow$

$\sin\theta - \cos\theta\sin\theta = 0 \Rightarrow \sin\theta(1 - \cos\theta) = 0$

$\sin\theta = 0 \Rightarrow \theta = 0, \pi$

$1 - \cos\theta = 0 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$

Xviii. $\sin 2\theta = \cos\theta \Rightarrow 2\sin\theta\cos\theta = \cos\theta \Rightarrow 2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$ $\cos\theta = 0 \Rightarrow \theta = \pi/2, 3\pi/2$

$2\sin\theta - 1 = 0 \Rightarrow 2\sin\theta = 1 \Rightarrow \sin\theta = 1/2 \Rightarrow \theta = \pi/6, 5\pi/6$

Xix. $\sin 2\theta - \cos 4\theta = 0 \Rightarrow \sin 2\theta - (1 - 2\sin^2(2\theta)) = 0 \Rightarrow$

$2\sin^2(2\theta) + \sin 2\theta - 1 = 0 \Rightarrow (2\sin 2\theta - 1)(\sin 2\theta + 1) = 0$

$2\sin 2\theta - 1 = 0 \Rightarrow 2\sin 2\theta = 1 \Rightarrow \sin 2\theta = 1/2$ $\pi/6$

$2\theta = \pi/6, 5\pi/6, 13\pi/6, 17\pi/6 \Rightarrow \theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$

$\sin 2\theta + 1 = 0 \Rightarrow \sin 2\theta = -1 \Rightarrow 2\theta = 3\pi/2, 7\pi/2 \Rightarrow \theta = 3\pi/4, 7\pi/4$

xx. $\csc^2\theta = \cot\theta + 1 \Rightarrow 1 + \cot^2\theta = \cot\theta + 1 \Rightarrow \cot^2\theta - \cot\theta = 0$

$\cot\theta(\cot\theta - 1) = 0$

$\cot\theta = 0 \Rightarrow \theta = \pi/2, 3\pi/2$

$\cot\theta - 1 = 0 \Rightarrow \cot\theta = 1 \Rightarrow \pi/4, 5\pi/4$

xxi. $2\cos^2\theta - 7\cos\theta - 4 = 0$

$(2\cos\theta + 1)(\cos\theta - 4) = 0$

$2\cos\theta + 1 = 0 \Rightarrow 2\cos\theta = -1 \Rightarrow \cos\theta = -\frac{1}{2} \xrightarrow{\pi/3} \theta = 2\pi/3, 4\pi/3$

$\cos\theta - 4 = 0 \Rightarrow \cos\theta = 4$ no solution

xxii. $\tan 2\theta + 2\cos\theta = 0 \Rightarrow \frac{\sin 2\theta}{\cos 2\theta} + 2\cos\theta = 0$

$\frac{2\sin\theta\cos\theta}{\cos 2\theta} + 2\cos\theta = 0 \Rightarrow 2\cos\theta\left(\frac{\sin\theta}{\cos 2\theta} + 1\right) = 0$

$2\cos\theta = 0 \Rightarrow \theta = \pi/2, 3\pi/2$

$\frac{\sin\theta}{1 - 2\sin^2\theta} = -1 \Rightarrow \sin\theta = -1 + 2\sin^2\theta \Rightarrow 2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0 \Rightarrow 2\sin\theta = -1 \Rightarrow \sin\theta = -\frac{1}{2} \xrightarrow{\pi/6} \theta = 7\pi/6, 11\pi/6$

$\sin\theta - 1 = 0 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \pi/2$

xxiii. $(\sin\theta + \cos\theta)^2 = (1)^2 \Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1 \Rightarrow$

$1 + 2\sin\theta\cos\theta = 1 \Rightarrow 2\sin\theta\cos\theta = 0 \Rightarrow$

$\sin\theta = 0 \rightarrow \theta = 0, \pi$

$\cos\theta = 0 \Rightarrow \theta = \pi/2, 3\pi/2$

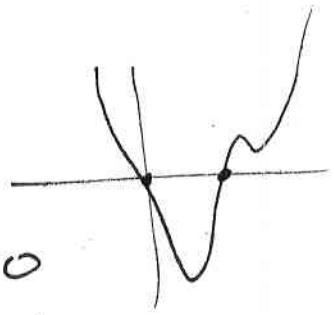
xxiv. $\cos(2\theta) + \sin^2\theta = 0 \Rightarrow 1 - 2\sin^2\theta + \sin^2\theta = 0 \Rightarrow$

$1 - \sin^2\theta = 0 \Rightarrow \sin^2\theta = 1 \Rightarrow \sin\theta = \pm 1$

$\Rightarrow \theta = \pi/2, 3\pi/2$

xxv. $x^2 - 2\sin 2x = 3x \Rightarrow x^2 - 3x - 2\sin(2x) = 0$

$x \approx 0, 2.2253$



xxvi. $6\sin x - e^x = 2 \Rightarrow 6\sin x - e^x - 2 = 0 \quad x > 0$

$x \approx .76215, 1.34878$