

Name KEY  
Math 268, Quiz #7, Spring 2012

1. Find the eigenvalues and eigenvectors of the matrix  $A = \begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$ .

$$\begin{vmatrix} 5-\lambda & 3 \\ -4 & 4-\lambda \end{vmatrix} = (5-\lambda)(4-\lambda) + 12 = 0$$

$$20 - 9\lambda + \lambda^2 + 12 = 0$$

$$\lambda^2 - 9\lambda + 32 = 0$$

$$\frac{9 \pm \sqrt{81 - 4(32)}}{2} = \frac{9 \pm \sqrt{81 - 128}}{2} = \frac{9 \pm \sqrt{-47}}{2} = \frac{9}{2} \pm \frac{\sqrt{47}}{2}i$$

$$\begin{bmatrix} 5 - \left(\frac{9}{2} + \frac{\sqrt{47}}{2}i\right) & 3 \\ -4 & 4 - \left(\frac{9}{2} + \frac{\sqrt{47}}{2}i\right) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - \frac{\sqrt{47}}{2}i & 3 \\ -4 & -\frac{1}{2} - \frac{\sqrt{47}}{2}i \end{bmatrix}$$

$$-4x_1 + \left(-\frac{1}{2} - \frac{\sqrt{47}}{2}i\right)x_2 = 0$$

$$-4x_1 = \left(\frac{1}{2} + \frac{\sqrt{47}}{2}i\right)x_2$$

$$x_1 = \left(-\frac{1}{8} - \frac{\sqrt{47}}{8}i\right)x_2$$

$$v_1 = \begin{bmatrix} 1 + \sqrt{47}i \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \end{bmatrix} + \begin{bmatrix} \sqrt{47} \\ 0 \end{bmatrix}i$$

$$v_2 = \begin{bmatrix} 1 \\ -8 \end{bmatrix} - \begin{bmatrix} \sqrt{47} \\ 0 \end{bmatrix}i$$