Name KEY

Math 285, Exam #2 Make-up Question, Spring 2012

Instructions: Show all work. You may use a TI-89 to check your answers, but points will be deducted if you obtain answers solely from your calculator. Be sure to answer all parts of all the questions, and use exact answers unless directed to round, or in word problems.

1. A mass of 5 kg stretches a spring 15 cm. The mass is acted on by an external force of 20cos(2*t*). (15 points)
   a. If the mass is set in motion from its equilibrium position with an initial velocity of 10 cm/sec, formulate the initial value problem describing the motion of the mass.

$m=5$

$5\cdot 98 = k\cdot 15 \Rightarrow \frac{980}{3}=k$

$\boxed{\begin{aligned} y(0)&=0 \\ y'(0)&=.1\end{aligned}}$

$5y'' + \frac{980}{3}y = 20\cos 2t$

$\boxed{y'' + \frac{196}{3}y = 4\cos 2t}$

   b. Solve the equation you found in part a.

$r^2 + \frac{196}{3} = 0$

$r = \pm\sqrt{\frac{196}{3}} = \pm\frac{14}{\sqrt{3}}i$

$Y(t) = A\cos 2t + B\sin 2t$

$Y'(t) = -2A\sin 2t + 2B\cos 2t$

$Y''(t) = -4A\cos 2t - 4B\sin 2t$

$y_c = C\cos\left(\frac{14}{\sqrt{3}}t\right) + D\sin\left(\frac{14}{\sqrt{3}}t\right)$

$-4A\cos 2t \cancel{- 4B\sin 2t} + \frac{196}{3}(A\cos 2t \cancel{+ B\sin 2t}) = 4\cos 2t$

$B=0$

$-4A + \frac{196}{3}A = 4 \qquad \frac{154}{3}A = 4 \qquad A = \frac{6}{77}$

$y(t) = C\cos\left(\frac{14}{\sqrt{3}}t\right) + D\sin\left(\frac{14}{\sqrt{3}}t\right) + \frac{6}{77}\cos 2t$

$0 = C + 0 + \frac{6}{77}$

$C = -\frac{6}{77}$

$y'(t) = \cancel{+\frac{6}{77}\cdot\frac{14}{\sqrt{3}}\sin\left(\frac{14}{\sqrt{3}}t\right)} + \frac{14}{\sqrt{3}}D\cos\left(\frac{14}{\sqrt{3}}t\right) \cancel{-\frac{12}{77}\sin 2t}$

$\frac{1}{10} = \frac{14}{\sqrt{3}}D \qquad D = \frac{\sqrt{3}}{140}$

$\boxed{y(t) = -\frac{6}{77}\cos\left(\frac{14}{\sqrt{3}}t\right) + \frac{\sqrt{3}}{140}\sin\left(\frac{14}{\sqrt{3}}t\right) + \frac{6}{77}\cos 2t}$

c. Describe the damping of the system: undamped, underdamped, critically damped or overdamped.

undamped

d. What is the transient solution (if one exists) and what is the steady state solution?

no transient solution since no dampening

$y(t)$ is steady state

e. Does the solution achieve resonance or does it contain beats? (Or neither.)

no resonance

it does contain beats

f. How does the natural frequency (without damping) compare to the quasi-frequency (if one exists)?

no quasi-frequency here

but dampening would slow oscillations

g. What is the long-term behaviour of the system as t→∞? If y approaches a numerical value in the limit, give it, or describe the behaviour of system.

the system will continue to oscillate.

h. Write the solution to the system (the homogeneous portion only) as $y = R\cos(\omega t - \delta)$ and clearly state the amplitude, the period (or quasi-period), the phase shift.

$A = \frac{-6}{77} \quad B = \frac{\sqrt{3}}{140}$

$R = \sqrt{\left(\frac{6}{77}\right)^2 + \left(\frac{\sqrt{3}}{140}\right)^2} \approx .07889 = \text{amplitude}$

$\omega = \frac{14}{\sqrt{3}}$

$T = \frac{2\pi \cdot \sqrt{3}}{14} = \frac{\pi\sqrt{3}}{7}$ period

$\tan^{-1} \frac{\frac{\sqrt{3}}{140}}{\frac{-6}{77}} = \tan^{-1}\left(\frac{\sqrt{3}\cdot 77}{-6\cdot 140}\right) \approx -.1574$ phaseshift

$y = .07889 \cos\left(\frac{14}{\sqrt{3}}t + .1574\right)$

i. State the values of the first 4 times y=0 (if fewer than that, state them all for t>0). You may do this numerically in your calculator.

$t_1 = .60708$

$t_2 = 1.00608$

$t_3 = 1.2322$

$t_4 = 1.851923$