Math 2255 Differential Equations Homework 5 Key

1. a. $y^{iv} + 4y''' + 3y = t.$

$p(t) = 4 \quad g(t) = t$

Solutions exist for all y & t

b. $y''' + ty'' + t^2y' + t^3y = \ln t$

Solutions exist for all $t > 0$

2. a. $y''' + y' = 0 \qquad r^3 + r = 0 \Rightarrow r(r^2+1) = 0 \quad r = 0,\ r = \pm i$

$y_c(t) = c_1 e^{0t}(=1) + c_2 \sin t + c_3 \cos t$ ✓

$$W = \begin{vmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ 0 & \cos t & -\sin t \end{vmatrix} = 1 \begin{vmatrix} -\sin t & \cos t \\ \cos t & -\sin t \end{vmatrix} = \sin^2 t - \cos^2 t = -\cos 2t \neq 0$$

for general t

it is a fundamental set

b. $y^{iv} + 2y''' + y'' = 0 \Rightarrow r^4 + 2r^3 + r^2 = 0 \Rightarrow r^2(\underbrace{r^2 + 2r + 1}_{(r+1)^2}) = 0$

$\Rightarrow r = 0$ repeated, $r = -1$ repeated

$y_c(t) = c_1 e^{(0)t}(=1) + c_2 t e^{(0)t}(=t) + c_3 e^{-t} + c_4 t e^{-t}$ ✓

$$W = \begin{vmatrix} 1 & t & e^{-t} & te^{-t} \\ 0 & 1 & -e^{-t} & e^{-t} - te^{-t} \\ 0 & 0 & e^{-t} & -e^{-t} - e^{-t} + te^{-t} = -2e^{-t} + te^{-t} \\ 0 & 0 & -e^{-t} & 2e^{-t} + e^{-t} - te^{-t} = 3e^{-t} - te^{-t} \end{vmatrix} =$$

$$(1)(1)\begin{vmatrix} e^{-t} & -2e^{-t} + te^{-t} \\ -e^{-t} & 3e^{-t} - te^{-t} \end{vmatrix} = 3e^{-2t} - \cancel{te^{-2t}} - 2e^{-2t} + \cancel{te^{-2t}} = e^{-2t}$$ ✓

it is a fundamental set

2c. $x^3y''' + x^2y'' - 2xy' + 2y = 0$ $\qquad$ $y = x^n \quad y' = nx^{n-1}$ $\qquad$ ②

$y'' = (n^2-n)x^{n-2} \quad y''' = (n^3-3n^2+2n)x^{n-3}$

$n^3-3n^2+2n + n^2-n - 2n + 2 = 0$

$n^3 - 2n^2 - n + 2 = 0$

$n^2(n-2) - 1(n-2) = 0$

$(n-2)(n^2-1) = 0 \Rightarrow (n-2)(n+1)(n-1) = 0 \qquad n = 2, 1, -1$

$\Rightarrow y_c(x) = c_1 x + c_2 x^2 + c_3 \frac{1}{x}$ ✓

$$W = \begin{vmatrix} x & x^2 & \frac{1}{x} \\ 1 & 2x & -\frac{1}{x^2} \\ 0 & 2 & \frac{2}{x^3} \end{vmatrix} = x \begin{vmatrix} 2x & -\frac{1}{x^2} \\ 2 & \frac{2}{x^3} \end{vmatrix} - 1 \begin{vmatrix} x^2 & \frac{1}{x} \\ 2 & \frac{2}{x^3} \end{vmatrix} =$$

$$x\left(\frac{4}{x^2} + \frac{2}{x^2}\right) - 1\left(\cancel{\frac{2}{x} - \frac{2}{x}}\right) = \frac{6x}{x^2} = \frac{6}{x} \checkmark \neq 0$$

it is a fundamental set

d. $y^{IV} - y = 0 \Rightarrow r^4 - 1 = 0 \qquad r = \pm 1, \pm i$

$y_c(t) = c_1 e^t + c_2 e^{-t} + c_3 \cos t + c_4 \sin t$ or

$c_1 \cosh t + c_2 \sinh t + c_3 \cos t + c_4 \sin t$ ✓

$$W = \begin{vmatrix} \cosh t & \sinh t & \cos t & \sin t \\ \sinh t & \cosh t & -\sin t & \cos t \\ \cosh t & \sinh t & -\cos t & -\sin t \\ \sinh t & \cosh t & \sin t & -\cos t \end{vmatrix} =$$

$$\cosh t \begin{vmatrix} \cosh t & -\sin t & \cos t \\ \sinh t & -\cos t & -\sin t \\ \cosh t & \sin t & -\cos t \end{vmatrix} - \sinh t \begin{vmatrix} \sinh t & \cos t & \sin t \\ \sinh t & -\cos t & -\sin t \\ \cosh t & \sin t & -\cos t \end{vmatrix} +$$

$$\cosh t \begin{vmatrix} \sinh t & \cos t & \sin t \\ \cosh t & -\sin t & \cos t \\ \cosh t & \sin t & -\cos t \end{vmatrix} - \sinh t \begin{vmatrix} \sinh t & \cos t & \sin t \\ \cosh t & -\sin t & \cos t \\ \sinh t & -\cos t & -\sin t \end{vmatrix}$$

d. cont'd

$$\begin{vmatrix} \cosh t & -\sin t & \cos t \\ \sinh t & -\cos t & -\sin t \\ \cosh t & \sin t & -\cos t \end{vmatrix} = \cosh t \begin{vmatrix} -\cos t & -\sin t \\ \sin t & -\cos t \end{vmatrix} - \sinh t \begin{vmatrix} -\sin t & \cos t \\ \sin t & -\cos t \end{vmatrix}$$

$$+ \cosh t \begin{vmatrix} -\sin t & \cos t \\ -\cos t & -\sin t \end{vmatrix} =$$

$$\cosh t \underbrace{[\cos^2 t + \sin^2 t]}_{=1} - \sinh t \underbrace{[\sin t \cos t - \sin t \cos t]}_{=0} + \cosh t \underbrace{[\sin^2 t + \cos^2 t]}_{=1}$$

$$= \cosh t + \cosh t = 2\cosh t$$

$$\begin{vmatrix} \sinh t & \cos t & \sin t \\ \sinh t & -\cos t & -\sin t \\ \cosh t & \sin t & -\cos t \end{vmatrix} = \sinh t \begin{vmatrix} -\cos t & -\sin t \\ \sin t & -\cos t \end{vmatrix} - \sinh t \begin{vmatrix} \cos t & \sin t \\ \sin t & -\cos t \end{vmatrix}$$

$$+ \cosh t \begin{vmatrix} \cos t & \sin t \\ -\cos t & -\sin t \end{vmatrix} =$$

$$\sinh t \underbrace{[\cos^2 t + \sin^2 t]}_{=1} - \sinh t \underbrace{[-\cos^2 t - \sin^2 t]}_{=-1} + \cosh t \underbrace{[-\sin t \cos t + \sin t \cos t]}_{=0}$$

$$= \sinh t + \sinh t = 2\sinh t$$

$$\begin{vmatrix} \sinh t & \cos t & \sin t \\ \cosh t & -\sin t & \cos t \\ \cosh t & \sin t & -\cos t \end{vmatrix} = \sinh t \begin{vmatrix} -\sin t & \cos t \\ \sin t & -\cos t \end{vmatrix} - \cosh t \begin{vmatrix} \cos t & \sin t \\ \sin t & -\cos t \end{vmatrix} +$$

$$\cosh t \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} =$$

$$\sinh t \underbrace{[\sin t \cos t - \sin t \cos t]}_{=0} - \cosh t \underbrace{[-\cos^2 t - \sin^2 t]}_{=-1} + \cosh t \underbrace{[\cos^2 t + \sin^2 t]}_{=1}$$

$$= \cosh t + \cosh t = 2\cosh t$$

$$\begin{vmatrix} \sinh t & \cos t & \sin t \\ \cosh t & -\sin t & \cos t \\ \sinh t & -\cos t & -\sin t \end{vmatrix} = \sinh t \begin{vmatrix} -\sin t & \cos t \\ -\cos t & -\sin t \end{vmatrix} - \cosh t \begin{vmatrix} \cos t & \sin t \\ -\cos t & -\sin t \end{vmatrix} + \sinh t \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix}$$

⑥

b. $(1-i)^{1/2}$ $1-i = \sqrt{2}\,e^{7\pi/4\, i} = \sqrt{2}\,e^{15\pi/4\, i}$

$= \sqrt[4]{2}\left(\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}\right),\ \sqrt[4]{2}\left(\cos\left(\frac{15\pi}{8}\right) + i\sin\left(\frac{15\pi}{8}\right)\right)$

6. a. $y''' + y' = 0$ $y(0)=0,\ y'(0)=1,\ y''(0)=2$

$r^3 + r = 0 \Rightarrow r(r^2+1) \Rightarrow$ $r=0,\ r=\pm 1$

$y_c(t) = c_1 + c_2\cos t + c_3 \sin t$

$0 = c_1 + c_2$

$y_c'(t) = -c_2 \sin t + c_3 \cos t$

$1 = c_3$

$y_c''(t) = -c_2\cos t - c_3 \sin t$

$2 = -c_2 \Rightarrow c_2 = -2 \Rightarrow c_1 = 2$

$y(t) = 2 - 2\cos t + \sin t$

b. $y^{IV} - 4y''' + 4y'' = 0$ $y(1)=-1,\ y'(1)=2,\ y''(1)=0,\ y'''(1)=0$

$r^4 - 4r^3 + 4r^2 = 0 \Rightarrow r^2(r-4r+4)=0 \Rightarrow r^2(r-2)^2=0$

$r=0$ repeated $r=2$ repeated

$y_c(t) = c_1 + c_2 t + c_3 e^{2t} + c_4 t e^{2t}$

$-1 = c_1 + c_2 + c_3 e^2 + c_4 e^2$

$y_c'(t) = c_2 + 2c_3 e^{2t} + c_4 e^{2t} + 2c_4 t e^{2t}$

$2 = c_2 + 2c_3 e^2 + c_4 e^2 + 2c_4 e^2 = c_2 + 2c_ce^2 + 3e_4 e^2$

$y_c''(t) = 4c_3 e^{2t} + 2c_4 e^{2t} + 2c_4 e^{2t} + 4c_4 t e^{2t} = 4c_3 e^{2t} + 4c_4 e^{2t} + 4k_4 t e^{2t}$

$0 = 4c_3 e^2 + 4c_4 e^2 + 4c_4 e^2 = 4c_3 e^2 + 8c_4 e^2$

$y_c'''(t) = 8c_3 e^{2t} + 8c_4 e^{2t} + 4c_4 e^{2t} + 8c_4 t e^{2t} = 8c_3 e^{2t} + 12c_4 e^{2t} + 8c_4 t e^{2t}$

$0 = 8c_3 e^2 + 12c_4 e^2 + 8c_4 e^2 = (8c_3 + 20c_4)e^2$

6b. cont'd

$$8c_3 + 20c_4 = 0 \qquad (4c_3 + 8c_4 = 0)\cdot 2$$
$$\underline{-8c_3 - 16c_4 = 0}$$
$$4c_4 = 0 \Rightarrow c_4 = 0,\ c_3 = 0$$

$$2 = c_2 + \cancel{2c_3e^{2}}^{0} + \cancel{3c_4e^{2}}^{0} \Rightarrow c_2 = 2$$

$$-1 = c_1 + c_2 + \cancel{c_3e^{2}}^{0} + \cancel{c_4e^{2}}^{0} \Rightarrow \begin{array}{c} -1 = c_1 + 2 \\ -2 \quad\ -2 \end{array} \Rightarrow c_1 = -3$$

$$y(t) = -3 + 2t$$

6c. $y^{iv} + 6y''' + 17y'' + 22y' + 14y = 0 \qquad y(0) = 1,\ y'(0) = -2,$

$r^4 + 6r^3 + 17r^2 + 22r + 14 = 0 \qquad y''(0) = 0,\ y'''(0) = 3$

$(r^2 + 2r + 2)(r^2 + 4r + 7) = 0$

$$r = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$$r = \frac{-4 \pm \sqrt{16-28}}{2} = \frac{-4 \pm 2\sqrt{3}i}{2} = -2 \pm \sqrt{3}i$$

$$y_c(t) = c_1e^{-t}\cos t + c_2e^{-t}\sin t + c_3e^{-2t}\cos\sqrt{3}t + c_4e^{-2t}\sin\sqrt{3}t$$

$$1 = c_1(1)(1) + c_2\cancel{(1)}(0) + c_3(1)(1) + \cancel{c_4(1)}(0) \Rightarrow 1 = c_1 + c_3$$

$$y_c'(t) = -c_1e^{-t}\cos t - c_1e^{-t}\sin t - c_2e^{-t}\sin t + c_2e^{-t}\cos t - 2c_3e^{-2t}\cos\sqrt{3}t$$
$$- \sqrt{3}c_3e^{-2t}\sin\sqrt{3}t - 2c_4e^{-2t}\sin\sqrt{3}t + \sqrt{3}c_4e^{-2t}\cos\sqrt{3}t$$

$$-2 = -c_1(1)(1) - \cancel{c_1(1)(0)} - \cancel{c_2(1)}(0) + c_2(1)(1) - 2c_3(1)(1) - \cancel{\sqrt{3}c_3(1)}(0)$$
$$-2\cancel{c_4(1)}(0) + \sqrt{3}c_4(1)(1)$$

$$-2 = -c_1 + c_2 - 2c_3 + \sqrt{3}c_4$$

$$y_c''(t) = \cancel{c_1e^{-t}\cos t} + 2c_1e^{-t}\sin t - \cancel{c_1e^{-t}\cos t} + \cancel{c_2e^{-t}\sin t} - 2c_2e^{-t}\cos t - \cancel{c_2e^{-t}\sin t}$$
$$+ 4c_3e^{-2t}\cos\sqrt{3}t + 4\sqrt{3}c_3e^{-2t}\sin\sqrt{3}t - 3c_3e^{-2t}\cos\sqrt{3}t +$$
$$4c_4e^{-2t}\sin\sqrt{3}t - 4\sqrt{3}c_4e^{-2t}\cos\sqrt{3}t - 3c_4e^{-2t}\sin\sqrt{3}t$$

6c cont'd

$$2c_1e^{-t}\sin t - 2c_2e^{-t}\cos t + c_3e^{-2t}\cos\sqrt{3}t + 4\sqrt{3}c_3e^{-2t}\sin\sqrt{3}t$$
$$+ c_4e^{-2t}\sin\sqrt{3}t - 4\sqrt{3}c_4e^{-2t}\cos\sqrt{3}t$$

$$0 = \cancel{2c_1(1)}\overset{0}{(0)} - 2c_2(1)(1) + c_3(1)(1) + \cancel{4\sqrt{3}c_3(1)(0)} + \cancel{c_4(1)(0)}$$
$$- 4\sqrt{3}c_4(1)(1)$$

$$0 = -2c_2 + c_3 - 4\sqrt{3}c_4$$

$$y_c'''(t) = -2c_1e^{-t}\sin t + 2c_1e^{-t}\cos t + 2c_2e^{-t}\cos t + 2c_2e^{-t}\sin t +$$
$$-2c_3e^{-2t}\cos\sqrt{3}t - \sqrt{3}c_3e^{-2t}\sin\sqrt{3}t - 8\sqrt{3}c_3e^{-2t}\sin\sqrt{3}t +$$
$$12c_3e^{-2t}\cos\sqrt{3}t - 2c_4e^{-2t}\sin\sqrt{3}t + 8\sqrt{3}c_4e^{-2t}\cos\sqrt{3}t + 12c_4e^{-2t}\sin\sqrt{3}t$$
$$\sqrt{3}c_4e^{-2t}\cos\sqrt{3}t$$

$$3 = -2\cancel{c_1(1)}(0) + 2c_1(1)(1) + 2c_2(1)(1) + 2\cancel{c_2(1)(0)} - 2c_3(1)(1)$$
$$- \sqrt{3}\cancel{c_3(1)}(0) - 8\sqrt{3}\cancel{c_3(1)}(0) + 12c_3(1)(1) - \cancel{2c_4(1)}\overset{0}{(0)} +$$
$$8\sqrt{3}c_4(1)(1) + \cancel{12c_4(1)}\overset{0}{(0)} + \sqrt{3}c_4(1)(1)$$

$$3 = 2c_1 + 2c_2 - 2c_3 + 12c_3 + 8\sqrt{3}c_4 + \sqrt{3}c_4$$
$$3 = 2c_1 + 2c_2 + 10c_3 + 9\sqrt{3}c_4$$

$$c_1 + c_3 = 1 \Rightarrow c_1 = 1 - c_3$$
$$-c_1 + c_2 - 2c_3 + \sqrt{3}c_4 = -2 \Rightarrow c_3 - 1 + c_2 - 2c_3 + \sqrt{3}c_4 = -2 \Rightarrow c_2 - c_3 + \sqrt{3}c_4 = -1$$
$$-2c_2 + c_3 - 4\sqrt{3}c_4 = 0$$
$$2c_1 + 2c_2 + 10c_3 + 9\sqrt{3}c_4 = 3 \Rightarrow 2 - 2c_3 + 2c_2 + 10c_3 + 9\sqrt{3}c_4 = 3 \Rightarrow$$
$$2c_2 + 8c_3 + 9\sqrt{3}c_4 = 1$$

$$c_2 - c_3 + \sqrt{3}c_4 = -1 \quad \times 2 = 2c_2 - 2c_3 + 2\sqrt{3}c_4 = -2 \Rightarrow -c_3 + 6\sqrt{3}c_4 = -2$$
$$-2c_2 + c_3 + 4\sqrt{3}c_4 = 0 \qquad \times 9$$
$$2c_2 + 8c_3 + 9\sqrt{3}c_4 = 1 \quad > \quad 9c_3 + 13\sqrt{3}c_4 = 1 \qquad 6\sqrt{3}c_4 + 2 = c_3$$
$$-9c_3 + 54\sqrt{3}c_4 = -18$$

$$67\sqrt{3}c_4 = -17 \Rightarrow c_4 = \frac{-17}{67\sqrt{3}} \qquad c_3 = \frac{-17}{67\sqrt{3}} \cdot 6\sqrt{3} + 2$$
$$c_3 = \frac{32}{67}$$

6c cont'd

$$c_2 = c_3 - \sqrt{3}c_4 - 1$$

$$\frac{32}{67} - \sqrt{3}\left(\frac{-17}{67\sqrt{3}}\right) - 1 = \frac{-18}{67}$$

$$c_1 = 1 - c_3 = 1 - \frac{32}{67} = \frac{35}{67}$$

$$y(t) = \frac{35}{67}e^{-t}\cos t - \frac{18}{67}e^{-t}\sin t + \frac{32}{67}e^{-2t}\cos\sqrt{3}t - \frac{17}{67\sqrt{3}}e^{-2t}\sin\sqrt{3}t$$

7a. $y''' + 4y' = t \qquad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1$

$$r^3 + 4r = 0 \Rightarrow r(r^2+4) = 0 \qquad r = 0, \pm 2i$$

$$y_c(t) = c_1 + c_2\cos 2t + c_3\sin 2t$$

$$Y(t) = (At+B)t = At^2 + Bt \qquad Y'(t) = 2At + B$$

this by itself can't give at since y' is last derivative

$$Y''(t) = 2A, \quad Y'''(t) = 0$$

$$0 + 4(2At+B) = t \Rightarrow 8At + 4B = t \Rightarrow 4B = 0 \quad B = 0$$

$$8A = 1 \Rightarrow A = \frac{1}{8}$$

$$y_p(t) = \frac{1}{8}t^2$$

$$y(t) = c_1 + c_2\cos 2t + c_3\sin 2t + \frac{1}{8}t^2$$

$$0 = c_1 + c_2 \qquad c_1 = -c_2$$

$$y'(t) = -2c_2\sin 2t + 2c_3\cos 2t + \frac{1}{4}t$$

$$0 = 2c_3 \Rightarrow c_3 = 0$$

$$y''(t) = -4c_2\cos 2t + \frac{1}{4}$$

$$1 = -4c_2 + \frac{1}{4} \Rightarrow \frac{3}{4} = -4c_2 \Rightarrow c_2 = -\frac{3}{16} \Rightarrow c_1 = \frac{3}{16}$$

$$y(t) = \frac{3}{16} - \frac{3}{16}\cos 2t + \frac{1}{8}t^2$$

0. $y^{iv} + 2y'' + y = 3t + 4 \qquad y(0) = 0,\ y'(0) = 0,\ y''(0) = 1,\ y'''(0) = 1$

$r^4 + 2r^2 + 1 = 0 \quad (r^2+1)^2 = 0 \qquad r = \pm i$ repeated

7b cont'd

$y_c(t) = c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t$

$Y(t) = At + B \quad Y'(t) = A \quad Y''(t) = Y'''(t) = Y^{iv}(t) = 0$

$0 + 2(0) + At + B = 3t + 4 \Rightarrow A = 3, B = 4$

$y_p(t) = 3t + 4$

$y(t) = c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t + 3t + 4$

$0 = c_1 + 4 \rightarrow c_1 = -4$

$y'(t) = 4 \sin t + c_2 \cos t + c_3 \cos t - c_3 t \sin t + c_4 t \cos t + c_4 \sin t + 3$

$0 = c_2 + c_3 + 3 \rightarrow c_2 + c_3 = -3$

$y''(t) = 4 \cos t \underbrace{- c_2 \sin t - c_3 \sin t - c_3 \sin t}_{-2c_3 \sin t} - c_3 t \cos t \underbrace{+ c_4 \cos t + c_4 \cos t}_{2c_4 \cos t} - c_4 t \sin t$

$1 = 4 + 2c_4 \Rightarrow -3 = -2c_4 \Rightarrow c_4 = \frac{3}{2}$

$y'''(t) = -4 \sin t - c_2 \cos t \underbrace{- 2c_3 \cos t - c_3 \cos t}_{-3c_3 \cos t} - c_3 t \sin t \underbrace{- 2c_4 \sin t - c_4 \sin t}_{-3c_4 \sin t} - c_4 t \cos t$

$1 = -\cancel{c_2} - 3c_3$

$-3 = \cancel{c_2} + c_3$

$-2 = -2c_3 \Rightarrow c_3 = 1 \Rightarrow c_2 + 1 = -3$

$c_2 = -4$

$y(t) = -4 \cos t - 4 \sin t + t \cos t - \frac{3}{2} t \sin t + 3t + 4$

8. $y''' - y'' + y' - y = \sec t \quad y(0) = 2, \ y'(0) = 1, \ y''(0) = 1$

$r^3 - r^2 + r - 1 = 0 \quad r^2(r-1) + (r-1) = 0 \quad (r-1)(r^2+1) = 0$

$r = 1, \pm i \quad y_1 = e^t, \ y_2 = \cos t, \ y_3 = \sin t$

$$W = \begin{vmatrix} e^t & \cos t & \sin t \\ e^t & -\sin t & \cos t \\ e^t & -\cos t & -\sin t \end{vmatrix} = e^t \begin{vmatrix} -\sin t & \cos t \\ -\cos t & -\sin t \end{vmatrix} - e^t \underbrace{\begin{vmatrix} \cos t & \sin t \\ -\cos t & -\sin t \end{vmatrix}}_{=0} + e^t \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix}$$

$$= e^t [\underbrace{\sin^2 t + \cos^2 t}_{=1}] + e^t [\underbrace{\cos^2 t + \sin^2 t}_{=1}] = 2e^t$$

$$W_1 = \begin{vmatrix} 0 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ 1 & -\cos t & -\sin t \end{vmatrix} = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t + \sin^2 t = 1$$

$$W_2 = \begin{vmatrix} e^t & 0 & \sin t \\ e^t & 0 & \cos t \\ e^t & 1 & -\sin t \end{vmatrix} = \begin{vmatrix} e^t & \sin t \\ e^t & \cos t \end{vmatrix} = e^t \sin t - e^t \cos t$$

$$W_3 = \begin{vmatrix} e^t & \cos t & 0 \\ e^t & -\sin t & 0 \\ e^t & -\cos t & 1 \end{vmatrix} = \begin{vmatrix} e^t & \cos t \\ e^t & -\sin t \end{vmatrix} = -e^t \sin t - e^t \cos t$$

$$Y(t) = e^t \int \frac{1 \cdot \sec t}{2e^t}\, dt + \cos t \int \frac{(\cancel{e^t} \sin t - \cancel{e^t} \cos t)\sec t}{2\cancel{e^t}}\, dt +$$

$$\sin t \int \frac{(-\cancel{e^t} \sin t - \cancel{e^t} \cos t)\sec t}{2\cancel{e^t}}\, dt =$$

$$\frac{1}{2}e^t \int e^{-t} \sec t\, dt + \frac{1}{2}\cos t \int \tan t - 1\, dt - \frac{1}{2}\sin t \int \tan t + 1\, dt$$

$$\underline{\frac{1}{2}e^t \int e^{-t} \sec t\, dt} + \frac{1}{2}\cos t\left[\ln|\sec t| - t + c_1\right] - \frac{1}{2}\sin t\left[\ln \sec t + t + c_2\right]$$

can't be integrated
by traditional techniques

9. $C = \frac{1}{4} \times 10^{-6}$, $R = 5000\,\Omega$, $L = 1$

$$Q'' + 5000 Q' + \frac{1}{\frac{1}{4}\times 10^{-6}} Q = 12 \qquad Q(0) = 0,\ Q'(0) = 0$$

$$Q'' + 5000 Q' + 4\times 10^6 Q = 12$$

$$r^2 + 5000 r + 4\times 10^6 = 0 \qquad r = \frac{-5000 \pm \sqrt{25\times 10^6 - 16\times 10^6}}{2} = \frac{-5000 \pm \sqrt{9\times 10^6}}{2}$$

$$r = \frac{-5000 \pm 3\times 10^3}{2} = \frac{-5000 \pm 3000}{2} \Rightarrow \frac{-2000}{2}, \frac{-8000}{2} = -1000, -4000$$

$$Y_c(t) = c_1 e^{-1000t} + c_2 e^{-4000t}$$

$$Y(t) = A \qquad Y'(t) = Y''(t) = 0 \qquad A \times 4\times 10^6 = 12 \Rightarrow$$

$$A = 3\times 10^{-6}$$

9 cont'd

(12)

$y(t) = c_1 e^{-1000t} + c_2 e^{-4000t} + 3\times10^{-6}$

$0 = c_1 + c_2 + 3\times10^{-6} \Rightarrow c_1 + c_2 = -3\times10^{-6}$

$y'(t) = -1000 c_1 e^{-1000t} - 4000 c_2 e^{-4000t}$

$0 = -1000 c_1 - 4000 c_2$

$-3\times10^{-3} = 1000 c_1 + 1000 c_2$

$\frac{-3\times10^{-3}}{-3000} = -3000 c_2 \qquad c_2 = 10^{-6}$

$c_1 = -3\times10^{-6} - 10^{-6} = -4\times10^{-6}$

$y(t) = -4\times10^{-6} e^{-1000t} + 10^{-6} e^{-4000t}$

$t = .001$ sec

$y(.001) = 1.54679\ldots\times10^{-6}$

$t = .01$ sec

$y(.01) = 2.99998\times10^{-6}$

$\lim_{t\to\infty} y(t) = 3\times10^{-6}$

10. a. beats – no damping, period of forcing similar to homogeneous solution

$y'' + 196y = \cos 15t$

b. resonance – period of forcing identical to homogeneous solution

$y'' + 196y = \cos 14t$

c. Asymptotically approaches zero → distinct roots, negative real part

10 d. contains a transient solution

$$y'' + 4y' + 3y = \cos 2t$$

e. oscillating steady state solution – forcing term w/ $\sin t$ or $\cos t$

$$y'' + 4y' + 3y = \cos 2t$$

f. no damping

$$y'' + 196y = 0$$

g. critical damping – repeated roots

$$y'' + 2y' + y = 0$$