Math 2255, Quiz #11, Spring 2013 Name KEY

Instructions: Show all work. You may use the attached tables for the transform formulas.

1. Find the Laplace Transforms for the following functions. You may use the definition or the table of formulas on back (I suggest the latter!).
   a. $\mathcal{L}\{3\cos(7t)\} = \dfrac{3s}{s^2+49}$

   b. $\mathcal{L}\left\{-\frac{1}{2}t^4e^{-2t}\right\} = -\frac{1}{2}\cdot\dfrac{4!}{(s+2)^5} = \dfrac{-12}{(s+2)^5}$

   c. $\mathcal{L}\{6\sinh(\pi t)\} = \dfrac{6\pi}{s^2-\pi^2}$

   d. $\mathcal{L}\{3t\cosh(t)\} = 3(-1)^1\dfrac{d}{ds}\left[\dfrac{s}{s^2-1}\right] = -3\left[\dfrac{s^2-1-s(2s)}{(s^2-1)^2}\right] = \dfrac{3s^2+3}{(s^2-1)^2}$

   $s^2-1-2s^2 = -s^2-1$

   e. $\mathcal{L}\{4u_3(t)-2u_5(t)\} = \dfrac{4e^{-3s}}{s} - \dfrac{2e^{-5s}}{s}$

2. Find the inverse Laplace transform of the following functions.
   a. $\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2+4}\right\} = \mathcal{L}^{-1}\left\{\dfrac{2s}{s^2+4} - \dfrac{1}{s^2+4}\right\} = 2\cos 2t - \frac{1}{2}\sin 2t$

   b. $\mathcal{L}^{-1}\left\{\frac{4s}{s^2-6s+7}\right\} = \mathcal{L}^{-1}\left\{\dfrac{4s}{(s^2-6s+9)-2}\right\} = \mathcal{L}^{-1}\left\{\dfrac{4s}{(s-3)^2-2}\right\} = \mathcal{L}^{-1}\left\{\dfrac{4(s-3)+12}{(s-3)^2-2}\right\}$

   $= 4e^{3t}\cosh\sqrt{2}t + \dfrac{12}{\sqrt{2}}e^{3t}\sinh\sqrt{2}t$

   c. $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2}\right\} = (t-2)u(t-2)$ or $(t-2)u_2(t)$

   d. $\mathcal{L}^{-1}\left\{\frac{3s-7}{s^2+4s+8}\right\} = \mathcal{L}^{-1}\left\{\dfrac{3(s+2)-13}{(s+2)^2+4}\right\} = \mathcal{L}^{-1}\left\{\dfrac{3(s+2)}{(s+2)^2+4} - \dfrac{13}{(s+2)^2+4}\right\} =$

   $3s-7 = 3(s+2)-7-6$; $s^2+4s+8 = s^2+4s+4+4$

   $3e^{-2t}\cos 2t - \frac{13}{2}e^{-2t}\sin 2t$

   e. $\mathcal{L}^{-1}\left\{\frac{10}{(s-4)^7}\right\} = \frac{10}{720}\mathcal{L}^{-1}\left\{\dfrac{6!}{(s-4)^7}\right\} = \frac{1}{72}t^6e^{4t}$