Math 2255, Quiz #13, Spring 2013 Name KEY

Instructions: Show all work. Use exact answers unless specified otherwise.

1. Solve the integro-differential equation $y'(t) + y(t) = 1 - \sin(t) + \int_0^t y(\tau)\cos(t-\tau)d\tau$ in terms of convolutions using Laplace transforms.

assume $y(0) = 1$

$$sY(s) - 1 + Y(s) = \frac{1}{s} - \frac{1}{s^2+1} + Y(s)\cdot\frac{s}{s^2+1}$$

$$sY(s) + Y(s) - Y(s)\frac{s}{s^2+1} = \frac{1}{s} - \frac{1}{s^2+1} + 1$$

$$Y(s)\left(s+1-\frac{s}{s^2+1}\right) = Y(s)\left[\frac{(s+1)(s^2+1)-s}{s^2+1}\right] = \frac{s^2+1-s+s^3+s}{s(s^2+1)}$$

$$Y(s)\left[\frac{s^3+s^2+1}{s^2+1}\right] = \frac{s^3+s^2+1}{s(s^2+1)}$$

$$Y(s) = \frac{s^3+s^2+1}{s(s^2+1)}\cdot\frac{s^2+1}{s^3+s^2+1} = \frac{1}{s}$$

$$y(t) = 1$$