Math 2255, Quiz #14, Spring 2013 Name KEY

Instructions: Show all work. Give exact answers whenever possible.

1. Use the method of series solutions to solve the differential equation $y'' - 9y = 0$. Find the equation for a_n, and give at least 4 terms. If even and odds have separate solutions, list at least 4 of each. This solution should produce a common series. Which one (or two) is it?

$$y(x) = \sum_{n=0}^{\infty} a_n x^n \qquad y' = \sum_{n=1}^{\infty} a_n n x^{n-1} \qquad y'' = \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2}$$

$$\sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} - 9\sum_{n=0}^{\infty} a_n x^n = \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2} - \sum_{n=0}^{\infty} 9a_n x^n =$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n - \sum_{n=0}^{\infty} 9a_n x^n =$$

$$\sum_{n=0}^{\infty} \left[a_{n+2}(n+2)(n+1) - 9a_n\right] x^n = 0$$

$$\frac{a_{n+2}\cancel{(n+2)}\cancel{(n+1)}}{\cancel{(n+2)}\cancel{(n+1)}} = \frac{9a_n}{(n+2)(n+1)}$$

$$a_{n+2} = \frac{9a_n}{(n+2)(n+1)}$$

$$a_2 = \frac{9a_0}{2\cdot 1} \qquad a_3 = \frac{9a_1}{3\cdot 2}$$

$$a_4 = \frac{9a_2}{4\cdot 3} = \frac{9^2 a_0}{4\cdot 3\cdot 2\cdot 1} \qquad a_5 = \frac{9a_3}{5\cdot 4} = \frac{9^2 a_1}{5\cdot 4\cdot 3\cdot 2}$$

$$a_6 = \frac{9a_4}{5\cdot 6} = \frac{9^3 a_0}{6!} = \frac{3^6 a_0}{6!} \qquad a_7 = \frac{9a_5}{7\cdot 6} = \frac{9^3 a_1}{7!} = \frac{3^6 a_1}{7!}$$

evens. odds

$$a_0 \sum_{n=0}^{\infty} \frac{3^{2n} x^{2n}}{(2n)!} = (3x)^{2n} \qquad a_1 \sum_{n=0}^{\infty} \frac{3^{2n} x^{2n+1}}{(2n+1)!} \quad \frac{1}{3}(3x)^{2n+1}$$

$a_0 \cosh(3x)$

$\frac{a_1}{3} \sinh(3x)$

$$y(x) = a_0\left[1 + \frac{9}{2}x^2 + \frac{81}{24}x^4 + \frac{729}{720}x^6 + \dots\right] + a_1\left[x + \frac{9}{6}x^3 + \frac{81}{120}x^5 + \frac{729}{5040}x^7 \dots\right]$$