Math 2255, Quiz #3, Spring 2013 Name KEY

Instructions: Show all work. Be sure to answer all parts of each question. Use exact answers unless specifically asked to round.

1. Solve the differential equation $y' = \frac{x^2-y^2}{xy}$. homogeneous, degree 2

$y = vx, \quad y' = v'x + v$

$v'x + v = \frac{x^2 - v^2x^2}{vx^2} = \frac{\cancel{x^2}(1-v^2)}{\cancel{x^2}(v)} \qquad v'x = \frac{1-v^2}{v} - v\left(\frac{v}{v}\right) = \frac{1-2v^2}{v}$

$\int \frac{v}{1-2v^2}\,dv = \int \frac{1}{x}\,dx$

$-\frac{1}{4}\ln|1-2v^2| = \ln x + C \Rightarrow \cancel{\ln} Ax = \cancel{\ln}\frac{1}{\sqrt[4]{1-2v^2}}$

$\frac{1}{Ax} = \sqrt[4]{1-2v^2} \Rightarrow \frac{1}{Ax^4} = 1-2v^2 \Rightarrow 2v^2 = 1 - \frac{A}{x^4} \Rightarrow$

$v^2 = \frac{1}{2} - \frac{A}{x^4} \Rightarrow v = \pm\sqrt{\frac{1}{2} - \frac{A}{x^4}}$

$\boxed{y = \pm x\sqrt{\frac{1}{2} - \frac{A}{x^4}}}$

2. Suppose a tank of water initially contains 50L of pure water. Salt water, with a concentration of 100g/L is pumped into the tank at a rate of 2L/sec. Suppose that the well-mixed solution is pumped out of the tank at the same rate.

a. Write a differential equation that models this situation.

$R_{in} = \frac{100g}{\cancel{L}} \cdot \frac{2\cancel{L}}{sec} = 200\frac{g}{sec}$

$R_{out} = \frac{A}{50\cancel{L}} \cdot \frac{2\cancel{L}}{sec} = \frac{A}{25}\frac{g}{sec}$

$\frac{dA}{dt} = 200 - \frac{A}{25}$

$A(0) = 0$

b. Solve the differential equation in part a. (You may want to put work on the back.)

$\frac{dA}{dt} = \frac{-1}{25}(A - 5000) \quad \int \frac{dA}{A-5000} = \int -\frac{1}{25}\,dt \Rightarrow \ln|A-5000| = -\frac{1}{25}t + C$

$A - 5000 = Ae^{-1/25 t} \Rightarrow A(t) = 5000 + A_0 e^{-1/25 t} \qquad 0 = 5000 + A_0 e^0$

$A_0 = -5000$

$\boxed{A(t) = 5000 - 5000e^{-1/25 t}}$

c. What is the equilibrium amount of salt in the tank? How long will it take to achieve 99% of this concentration? You may round your answer here to two decimal places.

5000g.

$99\% \Rightarrow 4950$

$4950 = 5000 - 5000e^{-1/25 t}$

$\frac{-50}{-5000} = e^{-1/25 t}$

$\ln(.01)\cdot(-25) = t$

$t = 115.13$ sec. $\approx$ just under 2 minutes