Math 2255, Quiz #5, Spring 2013 Name KEY

Instructions: Show all work to receive full credit. Give exact answers unless specifically asked to round.

1. Solve the differential equations below.
   a. $y'' + 4y' + 12 = 0, y(0) = 3, y'(0) = 0$

$r^2 + 4r + 12 = 0 \qquad r = \frac{-4 \pm \sqrt{16-48}}{2} = \frac{-4 \pm 4\sqrt{2}i}{2} = -2 \pm 2\sqrt{2}i$

$y_1 = e^{-2t}\cos(2\sqrt{2}t) \qquad y_2 = e^{-2t}\sin(2\sqrt{2}t)$

$y_c(t) = c_1 e^{-2t}\cos(2\sqrt{2}t) + c_2 e^{-2t}\sin(2\sqrt{2}t) \qquad c_1 = 3$

$y'(t) = -6e^{-2t}\cos(2\sqrt{2}t) - 6\sqrt{2}e^{-2t}\sin(2\sqrt{2}t) - 2c_2 e^{-2t}\sin(2\sqrt{2}t) + 2\sqrt{2}c_2\cos(2\sqrt{2}t)$

$0 = -6 + 2\sqrt{2}c_2 \Rightarrow 6 = 2\sqrt{2}c_2 \Rightarrow c_2 = \frac{3}{\sqrt{2}}$

$$\boxed{y(t) = 3e^{-2t}\cos(2\sqrt{2}t) + \frac{3}{\sqrt{2}}e^{-2t}\sin(2\sqrt{2}t)}$$

   b. $y'' - 16y = 0$. Write the solution to this problem as hyberbolic trig functions.

$r^2 - 16 = 0 \qquad r = \pm 4$

$y_1 = e^{-4t}, y_2 = e^{4t}$ or $y_1 = \cosh(4t), y_2 = \sinh(4t)$

$$\boxed{y_c(t) = c_1\cosh(4t) + c_2\sinh(4t)}$$

2. Calculate the Wronskian of both problems above. Show that the solutions form a fundamental set.

a) $W = \begin{vmatrix} e^{-2t}\cos(2\sqrt{2}t) & e^{-2t}\sin(2\sqrt{2}t) \\ -2e^{-2t}\cos(2\sqrt{2}t) - 2\sqrt{2}e^{-2t}\sin(2\sqrt{2}t) & -2e^{-2t}\sin(2\sqrt{2}t) + 2\sqrt{2}e^{-2t}\cos(2\sqrt{2}t) \end{vmatrix}$

$= \cancel{-2e^{-4t}\cos(2\sqrt{2}t)\sin(2\sqrt{2}t)} + 2\sqrt{2}e^{-4t}\cos^2(2\sqrt{2}t) + \cancel{2e^{-4t}\cos(2\sqrt{2}t)\sin(2\sqrt{2}t)}$
$+ 2\sqrt{2}e^{-4t}\sin^2(2\sqrt{2}t) =$

$2\sqrt{2}e^{-4t}(\cos^2(2\sqrt{2}t) + \sin^2(2\sqrt{2}t)) = \boxed{2\sqrt{2}e^{-4t}} \neq 0$

b) $W = \begin{vmatrix} \cosh(4t) & \sinh(4t) \\ 4\sinh(4t) & 4\cosh(4t) \end{vmatrix} = 4\cosh^2(4t) - 4\sinh^2(4t) =$
$4(\cosh^2(4t) - \sinh^2(4t)) = 4 \neq 0$

both are fundamental sets.