Math 2568, Quiz #12, Spring 2013 Name KEY

Instructions: Show all work. Be sure to use exact answers, and complete all parts of all problems.

1. Find the eigenvalues and corresponding eigenvectors for each matrix below. Do the eigenvectors form a basis for the space?

   a. $A = \begin{bmatrix} 1 & -7 \\ -1 & 5 \end{bmatrix}$ $A-\lambda I = \begin{bmatrix} 1-\lambda & -7 \\ -1 & 5-\lambda \end{bmatrix} \Rightarrow \det(A-\lambda I) = (1-\lambda)(5-\lambda) - 7$

$= 5 - 5\lambda - \lambda + \lambda^2 - 7 = \lambda^2 - 6\lambda - 2$

$\lambda = \frac{6 \pm \sqrt{36+8}}{2} = \frac{6 \pm \sqrt{44}}{2} = \frac{6 \pm 2\sqrt{11}}{2} = 3 \pm \sqrt{11}$ $\quad \lambda_1 = 3+\sqrt{11}$, $\lambda_2 = 3-\sqrt{11}$

$\vec{v}_1$: $\begin{bmatrix} 1-(3+\sqrt{11}) & -7 \\ -1 & 5-(3+\sqrt{11}) \end{bmatrix} = \begin{bmatrix} -2-\sqrt{11} & -7 \\ -1 & 2-\sqrt{11} \end{bmatrix}$ $\quad -x_1 + (2-\sqrt{11})x_2 = 0$, $x_1 = (2-\sqrt{11})x_2$ $\quad \vec{v}_1 = \begin{bmatrix} 2-\sqrt{11} \\ 1 \end{bmatrix}$

$\vec{v}_2$: is the conjugate of v_1 $\Rightarrow \begin{bmatrix} 2+\sqrt{11} \\ 1 \end{bmatrix}$.

2 vectors in $\mathbb{R}^2$ is a basis for $\mathbb{R}^2$ (vectors are lin. indep.)

   b. $A = \begin{bmatrix} 6 & 3 & -8 \\ 3 & 0 & -2 \\ 4 & 1 & -4 \end{bmatrix} \Rightarrow |A-\lambda I| = \begin{vmatrix} 6-\lambda & 3 & -8 \\ 3 & 0-\lambda & -2 \\ 4 & 1 & -4-\lambda \end{vmatrix} =$

$(6-\lambda)\begin{vmatrix} -\lambda & -2 \\ 1 & -4-\lambda \end{vmatrix} - 3\begin{vmatrix} 3 & -2 \\ 4 & -4-\lambda \end{vmatrix} - 8\begin{vmatrix} 3 & -\lambda \\ 4 & 1 \end{vmatrix} =$

$(6-\lambda)[(-\lambda)(-4-\lambda)+2] - 3[3(-4-\lambda)+8] - 8[3+4\lambda] =$

$(6-\lambda)[\lambda^2+4\lambda+2] - 3[-12 - 3\lambda + 8] - 24 - 32\lambda =$

$6\lambda^2 - \lambda^3 + 24\lambda - 4\lambda^2 + 12 - 2\lambda + 36 + 9\lambda - 24 - 24 - 32\lambda =$

$-\lambda^3 + 2\lambda^2 - \lambda = 0 \quad -\lambda(\underbrace{\lambda^2 - 2\lambda + 1}_{(\lambda-1)^2}) = 0 \quad \lambda_1 = 0,\ \lambda = 1$ (twice) $\lambda_2 = 1, \lambda_3 = 1$

$\lambda = 0$ $\vec{v}_1$: rref $A = \begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 1 & -4/3 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 - 2/3\,x_3 = 0$, $x_2 - 4/3\,x_3 = 0$, $x_3 = x_3$ $\Rightarrow$ $x_1 = 2/3\,x_3$, $x_2 = 4/3\,x_3$, $x_3 = x_3$ $\Rightarrow \vec{v}_1 = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$

$\lambda = 1$ $A-\lambda I = \begin{bmatrix} 5 & 3 & -8 \\ 3 & -1 & -2 \\ 4 & 1 & -5 \end{bmatrix}$ rref $\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 - x_3 = 0$, $x_2 - x_3 = 0$, $x_3 =$ free $\Rightarrow$ $x_1 = x_3$, $x_2 = x_3$, $x_3 = x_3$ $\Rightarrow \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Only 2 vectors in eigenspace in $\mathbb{R}^3$
So not a basis for $\mathbb{R}^3$