Math 2568, Quiz #2, Spring 2013 Name KEY

Instructions: You must show all work to receive full credit for the problems below. You may check your work with a calculator, but answers without work will receive minimal credit. Use exact answers unless the problem starts with decimals or you are specifically asked to round.

1. Consider the system of equations $\begin{cases} \phantom{x_1 + {}} x_2 + 2x_3 = 5 \\ x_1 + 2x_2 + 3x_3 = 8 \\ 2x_1 \phantom{{}+2x_2} + 6x_3 = 11 \end{cases}$.
   a. Write the system of equations as a vector equation.
   b. Write the system of equation as a matrix equation.

a. $x_1\begin{bmatrix}0\\1\\2\end{bmatrix} + x_2\begin{bmatrix}1\\2\\0\end{bmatrix} + x_3\begin{bmatrix}2\\3\\6\end{bmatrix} = \begin{bmatrix}5\\8\\11\end{bmatrix}$

b. $\begin{bmatrix}0 & 1 & 2\\1 & 2 & 3\\2 & 0 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}5\\8\\11\end{bmatrix}$

2. Consider the matrix $A = \begin{bmatrix}1 & -2 & 7 & 5\\0 & 3 & 0 & 3\\1 & 2 & 0 & 2\\1 & 5 & -1 & 4\end{bmatrix}$. Do the vectors represented by the columns of the matrix span R^4? Why or why not? If they do, choose a random vector and prove it is a linear combination of the other vectors in the matrix and the multiples of each vector needed to obtain it. If they do not span R^4, find one vector outside the span and show that the system is inconsistent.

Yes, They do Span $\mathbb{R}^4$

when the matrix is fully row-reduced we obtain $\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 1\\0 & 0 & 0 & 0\end{bmatrix}$

this (non-augmented) matrix has only 3 pivots so only three of the vectors are independent. The 4th is in the span of the others so it only spans a 3D space. Consider a vector which is a linear combination of these vectors & then change one digit, e.g. $\begin{bmatrix}-1\\3\\3\\6\end{bmatrix}$ is in the span. try $\begin{bmatrix}-1\\3\\3\\7\end{bmatrix}$. Augment the matrix & reduce: $\left[\begin{array}{cccc|c}1 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 1 & 0\\0 & 0 & 1 & 1 & 0\\0 & 0 & 0 & 0 & 1\end{array}\right]$ inconsistent so this vector is not in the span of the others.