Math 2568, Quiz #8, Spring 2013 Name KEY

1. For the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 1 & 1 & 1 & 9 \end{bmatrix}$, find an explicit description of Nul A and Col A.

row reduces to $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\text{Col } A = \text{span}\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\} = \mathbb{R}^3$

Nul A:

$x_1 = -4x_4$
$x_2 = -5x_4$ $\Rightarrow$ $\vec{x} = t\begin{bmatrix}-4\\-5\\0\\1\end{bmatrix}$
$x_3 = 0$
$x_4 = x_4$ (free)

$\text{Nul } A = \text{span}\left\{\begin{bmatrix}-4\\-5\\0\\1\end{bmatrix}\right\}$

2. Determine if the following sets represent vector spaces.
   a. The set of even functions (i.e. all functions such that $f(x) = f(-x)$).

a) is the zero function $f(x)=0$ an even function? Yes.
b) if you add two even functions together, do you get an even function? yes.
c) if you multiply an even function by a scalar do you get an even function? yes.

It is a vector space.

   b. $W = \left\{\begin{bmatrix} a \\ b^2 \end{bmatrix}, a, b \text{ real}\right\}$

a) if $a=0, b=0$ $\begin{bmatrix}0\\0\end{bmatrix}$ is in the set

not a vector space

b) $\begin{bmatrix} a \\ b^2 \end{bmatrix} + \begin{bmatrix} c \\ d^2 \end{bmatrix} = \begin{bmatrix} a+c \\ b^2+d^2 \end{bmatrix}$ $a+c$ is real

$b^2+d^2 = (\sqrt{b^2+d^2})^2$ so $\sqrt{b^2+d^2}$ is real since b^2+d^2 is always positive

c) $k\begin{bmatrix} a \\ b^2 \end{bmatrix} = \begin{bmatrix} ka \\ kb^2 \end{bmatrix}$ ka is okay, its real

but if $k=-1$ $-1\begin{bmatrix}1\\4\end{bmatrix} = \begin{bmatrix}-1\\-4\end{bmatrix}$ but -4 is not the square of a real b