Multiple Variable Integration Review Key

1) $\int x \ln y \, dy = x \int \ln y \, dy$ $\quad u = \ln y \quad dv = dy$ $\quad du = \frac{1}{y} dy \quad v = y$

$= x\left[y \ln y - \int \frac{1}{\cancel{y}} \cdot \cancel{y} \, dy\right] = x\left[y \ln y - y\right] + C(x)$

2) $\int 3r + 2\theta \, dr = \frac{3}{2}r^2 + 2\theta r + C(\theta)$

3) $\int x + y \, dz = xz + yz + C(x,y)$

4) $\int e^{x+w} - \frac{1}{w} \, dw = \int e^w e^x - \frac{1}{w} \, dw = e^{x+w} - \ln w + C(x,y,z)$

5) $\int \frac{1}{(1+x^2)+y^2} \, dy = \frac{1}{\sqrt{1+x^2}} \arctan\left(\frac{y}{\sqrt{1+x^2}}\right) + C(x)$

$a^2 = 1+x^2$

$a = \sqrt{1+x^2}$

6) $\int x \sin(xy) \, dy = -\cos(xy) + C(x)$

$u = xy$

$du = x \, dy$

$\int \sin u \, du$

7) $\int x \sin(xy) \, dx$ $\quad u = x \quad dv = \sin(xy) dx$ $\quad du = dx \quad v = -\frac{1}{y}\cos(xy)$

$w = xy$

$dw = y\,dx \rightarrow \frac{1}{y} dw = dx$

$\int \frac{1}{y} \sin w \, dw = -\frac{1}{y} \cos w$

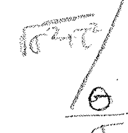
$-\frac{x}{y}\cos(xy) - \int -\frac{1}{y}\cos(xy)\,dx \longrightarrow$ Same substitution as above

$= -\frac{x}{y}\cos(xy) + \frac{1}{y^2}\sin(xy) + C(y)$

8) $\int \frac{1}{\sqrt{\sigma^2 + \tau^2}} \, d\tau$ $\quad \tau = \sigma \tan\theta \quad d\tau = \sigma \sec^2\theta \, d\theta$

$\sqrt{\sigma^2+\tau^2} = \sqrt{\sigma^2 + \sigma^2 \tan^2\theta} = \sqrt{\sigma^2(1+\tan^2\theta)}$ $\quad \frac{\tau}{\sigma} = \tan\theta$

$\sqrt{\sigma^2 \sec^2\theta} = \sigma \sec\theta$



$= \int \frac{\cancel{\sigma} \sec^2\theta \, d\theta}{\cancel{\sigma} \sec\theta} = \int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{\sigma^2+\tau^2}}{\sigma} + \frac{\tau}{\sigma}\right| + C(\sigma) = \ln\left|\frac{\sqrt{\sigma^2+\tau^2}+\tau}{\sigma}\right| + C(\sigma)$

9) $\int xyz\sqrt{4-x^2-y^2}\,dx$ $u = 4-x^2-y^2$

$du = -2x\,dx \Rightarrow -\frac{1}{2}du = x\,dx$

$\int -\frac{1}{2}yz\,u^{1/2}\,du = -\frac{1}{\cancel{2}}yz\cdot\frac{\cancel{2}}{3}u^{3/2} \Rightarrow$

$-\frac{1}{3}yz(4-x^2-y^2)^{3/2} + C(y,z)$

10) $\int \rho^2 \sin^2\varphi\,d\varphi = \frac{1}{2}\rho^2\int 1-\cos 2\varphi\,d\varphi = \frac{1}{2}\rho^2\left[\varphi - \frac{1}{2}\sin 2\varphi\right] + C(\rho)$

$= \frac{1}{2}\rho^2\varphi - \frac{1}{4}\rho^2\sin(2\varphi) + C(\rho)$

11) $\int \frac{1}{1-xy}\,dy$ $u = 1-xy$

$du = -x\,dy \Rightarrow -\frac{1}{x}du = dy$

$\int -\frac{1}{x}\cdot\frac{1}{u}\,du = -\frac{1}{x}\ln|u| \Rightarrow -\frac{1}{x}\ln|1-xy| + C(x)$