Math 1152, Quiz #11, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Determine whether the following sequences converge or diverge. Explain your reasoning and be sure to check that all the conditions of the test you apply are satisfied.

a. $\sum_{k=1}^{\infty} \frac{\sinh k}{k}$

using nth term/divergence test

$$\lim_{k\to\infty} \frac{\sinh k}{k} = \lim_{k\to\infty} \frac{e^k - e^{-k}}{2k} = \lim_{k\to\infty} \frac{e^k}{2k} - \cancel{\lim_{k\to\infty} \frac{e^{-k}}{2k}}^{\,0}$$

$$= \lim_{k\to\infty} \frac{e^k}{2k} = \infty \therefore \text{diverges}$$

b. $\sum_{n=1}^{\infty} \frac{1}{n}$

according to the integral test

$$\int_1^\infty 1/n \, dn = \ln n \Big|_1^\infty = -\cancel{\ln(1)}^{\,0} + \ln\infty = \infty$$

$\therefore$ diverges

(this is the harmonic series. it also diverges by the p-test).

c. $\sum_{k=0}^{\infty} e^{-k} = \sum_{k=0}^{\infty} (1/e)^k$

by the geometric series test $|1/e| < 1$ $\therefore$ this series converges to $\frac{1}{1-1/e} = \frac{e}{e-1}$

d. $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$ by the integral test $\int_0^\infty \frac{1}{1+n^2}\,dn =$

$\arctan n \Big|_0^\infty = \pi/2$ converges since the integral converges.