Math 1152, Quiz #12, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Determine whether the following sequences converge or diverge. Explain your reasoning and be sure to check that all the conditions of the test you apply are satisfied.

a. $\sum_{k=1}^{\infty} \frac{\cos n\pi}{\sqrt{n^2+1}}$ n

$\cos n\pi = (-1)^n$

$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} = 0$

$\therefore$ Converges by alternating series test

b. $\sum_{n=1}^{\infty} \frac{n2^n}{n!}$

$\lim_{n\to\infty} \left| \frac{(n+1)\,2^{n+1}}{(n+1)!} \cdot \frac{n!}{n\,2^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(n+1)}\,\cancel{2^n}\cdot 2}{\cancel{(n+1)}\,\cancel{2^n}\cdot n} \right| =$

$\lim_{n\to\infty} \frac{2}{n} = 0 < 1$ $\therefore$ Converges by ratio test

c. $\sum_{k=0}^{\infty} \frac{1}{2k^3+1}$ vs. $\sum_{k=1}^{\infty} \frac{1}{k^3}$ $\lim_{k\to\infty} \frac{1}{2k^3+1} \cdot \frac{k^3}{1} = \frac{1}{2}$

$\therefore$ since $\sum_{k=1}^{\infty} \frac{1}{k^3}$ converges by p-test, $\sum_{k=0}^{\infty} \frac{1}{2k^3+1}$ converges by limit comparison

d. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

$\int_1^{\infty} \frac{\ln x}{x}\,dx =$ $u = \ln x$, $du = 1/x\,dx$ $\int u\,du$

$\frac{1}{2}(\ln x)^2 \Big|_1^{\infty} = \infty$ diverges by integral test.