

**Instructions:** Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Find the power series representation for the function  $f(x) = \frac{2}{3-x} \cdot \frac{1}{3}$

$$\frac{2/3}{1 - \frac{x}{3}} \quad a = \frac{2}{3} \quad r = \frac{x}{3}$$

$$\sum_{k=0}^{\infty} \frac{2}{3} \left(\frac{x}{3}\right)^k$$

$$\lim_{k \rightarrow \infty} \left| \frac{\frac{2}{3} \left(\frac{x}{3}\right)^{k+1}}{\frac{2}{3} \left(\frac{x}{3}\right)^k} \right| = \left| \frac{x}{3} \right| < 1$$

$-3 < x < 3$   
diverges at both  $\pm 3$

2. What is  $f'(x)$ ? What is its power series representation? (Use your answer in Problem #1.)

$$\sum_{k=1}^{\infty} \frac{2}{3} k \left(\frac{x}{3}\right)^{k-1} \left(\frac{1}{3}\right) = \sum_{k=1}^{\infty} \frac{2}{3} k \frac{x^{k-1}}{3^k}$$

$$\lim_{k \rightarrow \infty} \left| \frac{\frac{2}{3} (k+1) \left(\frac{x}{3}\right)^k \frac{1}{3}}{\frac{2}{3} k \left(\frac{x}{3}\right)^{k-1} \left(\frac{1}{3}\right)} \right| =$$

$$\lim_{k \rightarrow \infty} \left(\frac{x}{3}\right) < 1$$

$-3 < x < 3$   
diverges at  $\pm 3$

3. What is  $\int f(x) dx$ ? What is the power series representation?

$$\sum_{k=0}^{\infty} \frac{2}{3} \frac{x^{k+1}}{3^k (k+1)} + C$$

$$\lim_{k \rightarrow \infty} \left| \frac{\frac{2}{3} x^{k+2} / 3^{k+1} (k+2)}{\frac{2}{3} x^{k+1} / 3^k (k+1)} \right| < 1$$

$\frac{x}{3} < 1$   
 $-3 < x < 3$   
↑  
diverges non-alt.

$\sum \frac{(-3)^{k+1}}{(3^k)(k+1)}$  conv. alternating

4. On what interval are all three functions defined on? Are they all the same?

$f(x)$ :  $(-3, 3)$   
 $f'(x)$ :  $(-3, 3)$   
 $\int f(x)$ :  $[-3, 3)$  ← includes endpoint

$>$  Same