

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Integrate.

$$a. \int \frac{(x^2+1)dx}{x(x^2-1)} = \int \frac{x^2+1}{x(x-1)(x+1)} dx = \int \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} dx$$

$$A \ln|x| + B \ln|x-1| + C \ln|x+1| + K$$

$$x^2+1 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

$$x=0 \Rightarrow 1 = A(-1)(1) \Rightarrow A = -1$$

$$x=1 \Rightarrow 2 = B(1)(2) \Rightarrow B = 1$$

$$x=-1 \Rightarrow 2 = C(-1)(-2) \Rightarrow C = 1$$

$$\Rightarrow \boxed{\ln|x| + \ln|x-1| + \ln|x+1| + C}$$

$$b. \int \frac{dx}{x^2+2x-3} = \int \frac{dx}{(x+3)(x-1)} = \int \frac{A}{x+3} + \frac{B}{x-1} dx$$

$$= A \ln|x+3| + B \ln|x-1| + C$$

$$A(x-1) + B(x+3) = 1$$

$$x=1 \Rightarrow B(4) = 1 \Rightarrow B = \frac{1}{4}$$

$$x=-3 \Rightarrow A(-4) = 1 \Rightarrow A = -\frac{1}{4}$$

$$\Rightarrow \boxed{-\frac{1}{4} \ln|x+3| + \frac{1}{4} \ln|x-1| + C}$$

2. Set up the partial fraction decomposition for the following expression:

$\frac{x^2+3}{x^3(x+1)(x-2)(x^2+4)^2(x^2+7x+5)}$. Do not attempt to solve for any of the variables, just set up the decomposition.

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x+1} + \frac{E}{x-2} + \frac{Fx+G}{x^2+4} + \frac{Hx+I}{(x^2+4)^2} + \frac{Jx+K}{x^2+7x+5}$$