Math 1152, Quiz #6, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. Verify that the solution to the differential equation $y'' + y = 0$ has the form $y = A\cos(x) + B\sin(x)$. Then use the initial conditions $y(0) = 4, y'(0) = -3$ to solve for the values of A and B.

$y' = -A\sin x + B\cos x \qquad y'' = -A\cos x - B\sin x$

$y'' + y = -A\cos x - B\sin x + A\cos x + B\sin x = 0$ ✓

$4 = A\cos(0) + \cancel{B\sin(0)} \qquad A = 4$

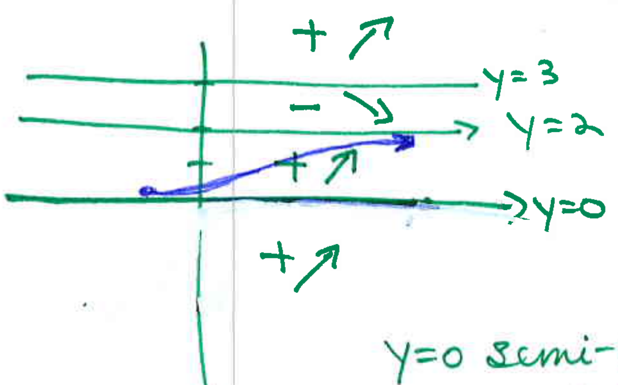
$-3 = \cancel{-A\sin(0)} + B\cos(0) \quad \Rightarrow \quad B = -3$

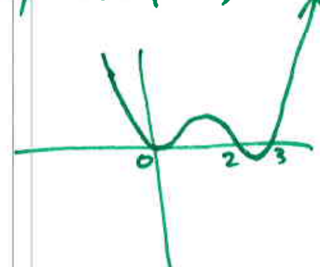
$y = 4\cos x - 3\sin x$

2. Sketch the main features of the direction field for $y' = y^4 - 5y^3 + 6y^2$. Indicate all equilibria, whether they are stable, unstable or semi-stable, and the sign of the slope in each region of the graph. Use that information to sketch the trajectory of a particle whose initial conditions are $y(-1) = 0.01$.

$y^2(y^2 - 5y + 6)$

$y^2(y-3)(y-2)$





y=0 semi-stable/mixed
y=2 stable/attracting
y=3 unstable/repelling

3. Use Euler's method to calculate 5 steps of the solution to the differential equation $y' = \frac{2x}{y^2-1}$, for a particle starting at the point $y(1) = 2$ with a step size of $\Delta x = 0.1$.

$m_1 = \frac{2(1)}{4-1} = \frac{2}{3} = .4$

$y(1.1) = .4(.1) + 2 = 2.04$

$m_2 = \frac{2(1.1)}{2.04^2-1} = \frac{2.2}{3.1616} = .69585$

$y(1.2) = .69585(.1) + 2.04 = 2.109585$

$m_3 = \frac{2(1.2)}{(2.109)^2-1} = \frac{2.4}{3.45} = .6955818$

$y(1.3) = .6955818(.1) + 2.109585 = 2.179143$

$m_4 = \frac{2(1.3)}{2.179^2-1} = \frac{2.6}{3.7486} = .69358$

$y(1.4) = .69358(.1) + 2.179143 = 2.2485$

$m_5 = \frac{2(1.4)}{2.24^2-1} = \frac{2.8}{4.055} = .69037669$

$y(1.5) = .69037669(.1) + 2.2485 = 2.3175$

$y(1.5) \approx 2.3175$