Math 2153, Quiz #12, Spring 2014 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the curvature of the vector-valued function $\vec{r}(t) = t^2\hat{\imath} + t\sin \pi t\,\hat{\jmath} + 2t\hat{k}$, at the point (1,0,2). Use the formula $\kappa = \frac{\|\vec{r}'\times\vec{r}''\|}{\|\vec{r}'\|^3}$.

$t=1$

$r'(t) = 2t\hat{\imath} + (\sin\pi t + t\pi\cos\pi t)\hat{\jmath} + 2\hat{k}$

$r''(t) = 2\hat{\imath} + (\pi\cos\pi t + \pi\cos\pi t - t\pi^2\sin\pi t)\hat{\jmath} =$

$2\hat{\imath} + (2\pi\cos\pi t - t\pi^2\sin\pi t)\hat{\jmath}$

$r'(1) = 2\hat{\imath} + -\pi\hat{\jmath} + 2\hat{k}$ $\quad r''(1) = 2\hat{\imath} + (-2\pi)\hat{\jmath}$

$\|r' \times r''\| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -\pi & 2 \\ 2 & -2\pi & 0 \end{vmatrix} = \|(+4\pi)\hat{\imath} - (-4)\hat{\jmath} + (-4\pi + 2\pi)\hat{k}\| =$

$\|4\pi\hat{\imath} + 4\hat{\jmath} + (-2\pi)\hat{k}\| =$

$\sqrt{16\pi^2 + 16 + 4\pi^2} = \sqrt{20\pi^2 + 16} = 2\sqrt{5\pi^2+4}$

$\|r'(1)\| = \sqrt{4 + \pi^2 + 4} = \sqrt{8+\pi^2}$

$K(1) = \dfrac{2\sqrt{5\pi^2+4}}{(8+\pi^2)^{3/2}} \approx .19338$

2. Find the surface area for the function $f(x,y) = 4 - x^2 - y^2$ above the plane.

$f_x = -2x \quad f_y = -2y \quad \iint_R \sqrt{1 + 4x^2 + 4y^2}\, dA$ $\quad x^2+y^2=4$, $r=2$

$\int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\; r\,dr\,d\theta$ $\quad u = 1+4r^2$, $\frac{du}{8} = \cancel{8} r\,dr$

$\int \frac{1}{8} u^{1/2}\, du$

$\frac{1}{\cancel{8}_4}\cdot\frac{\cancel{2}}{3} u^{3/2} = \frac{1}{12}(1+4r^2)^{3/2}\Big|_0^2 = \frac{1}{12}\left[(17)^{3/2} - 1\right]$

$\int_0^{2\pi} \frac{1}{12}(17^{3/2}-1)\,d\theta = \frac{\pi}{6}(17^{3/2}-1)$