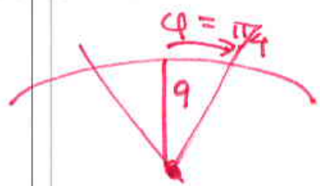
Math 2153, Quiz #9, Spring 2014 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the volume of the solid bounded above by the sphere of radius 9 centered at the origin, and below by the cone $z = \sqrt{x^2 + y^2}$. [Hint: it will be easier to integrate in spherical coordinates.]

$\rho\cos\varphi = \rho\sin\varphi$

$\Rightarrow \varphi = \pi/4$



$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^9 \rho^2\sin\varphi\, d\rho\, d\varphi\, d\theta$$

$$\frac{\rho^3}{3}\Big|_0^9 = 243$$

$$\int_0^{2\pi}\int_0^{\pi/4} 243\sin\varphi\, d\varphi\, d\theta = \int_0^{2\pi} -243\cos\varphi \Big|_0^{\pi/4} d\theta = \int_0^{2\pi} -243(1/\sqrt{2} - 1)\, d\theta$$

$$\left(243 - \frac{243}{\sqrt{2}}\right)\cdot 2\pi$$

2. Find the potential function, if it exists, for the vector field $F(x, y, z) = (3x^2y - z)\hat{\imath} + (yz + x^3)\hat{\jmath} + (\frac{1}{2}y^2 - x)\hat{k}$. If it does not exist, verify this by applying the test for conservative vector fields.

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2y - z & yz + x^3 & \frac{1}{2}y^2 - x \end{vmatrix} = \vec{0} \text{ (on quiz #8)}$$

$$\int 3x^2y - z\, dx = x^3y - xz + G(y,z)$$

$$\int yz + x^3\, dy = \frac{1}{2}y^2z + x^3y + H(x,z)$$

$$\int \frac{1}{2}y^2 - x\, dz = \frac{1}{2}y^2z - xz + I(x,y)$$

$$f(x,y,z) = x^3y - xz + \frac{1}{2}y^2z + K$$