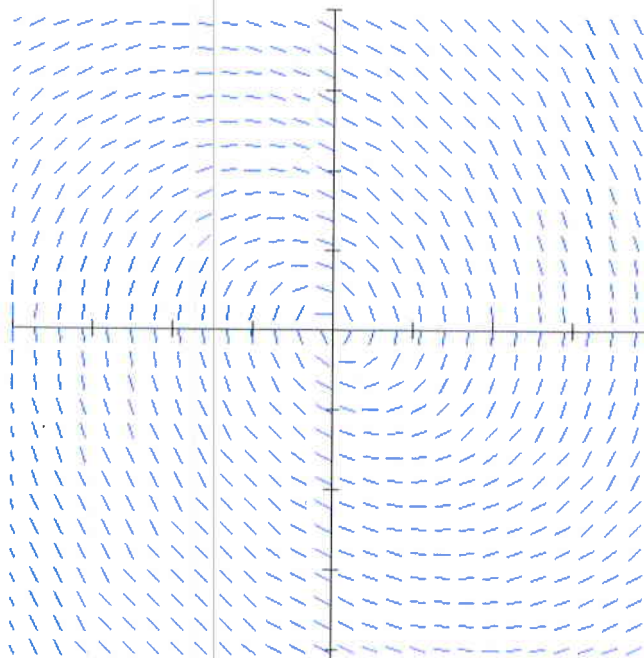


there is an apparent circular shape to the graph. Part of what makes this graph complicated is that there are vertical slopes at  $y=0$ . These are points where the solution curve breaks down and becomes undefined. We can plot trajectories up to these points, but since the horizontal axis represents time when we use  $t$  instead of  $x$  for the dependent variable, we cannot go back in time, and so cannot go leftward on the graph. If we use  $x$ , and this is a physical plane, we will end up with the swirling trajectory, since we will be able to go left.

The full direction field is plotted below.

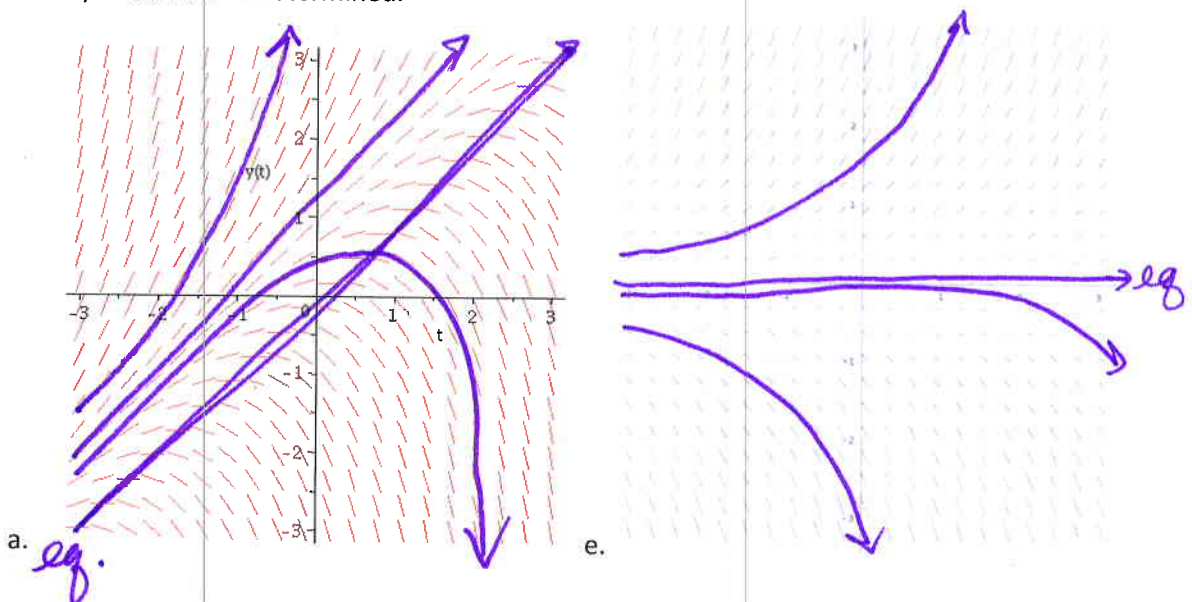
In other graphs, equilibria that are not horizontal lines can also change characteristics as they cross over a zero or a sign change, for instance, two halves of a parabolic-shaped equilibrium curve.

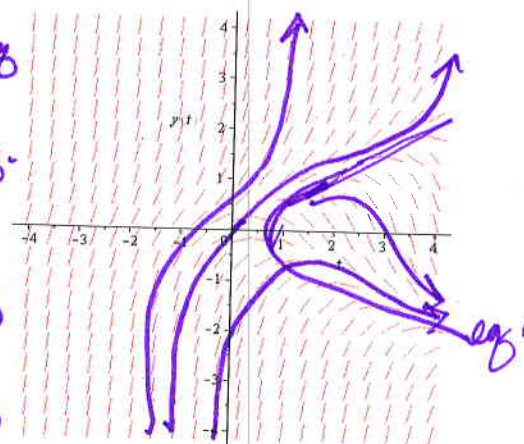
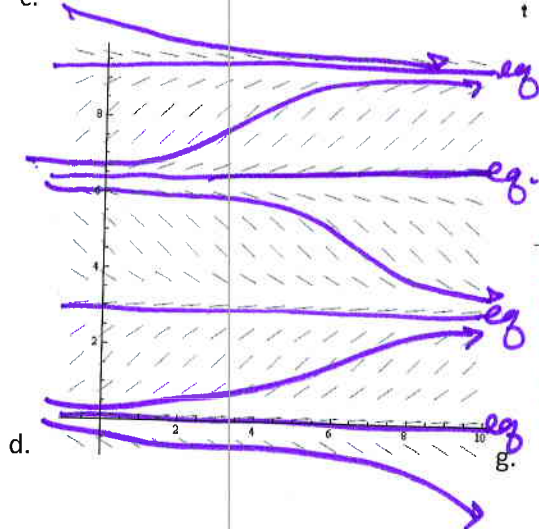
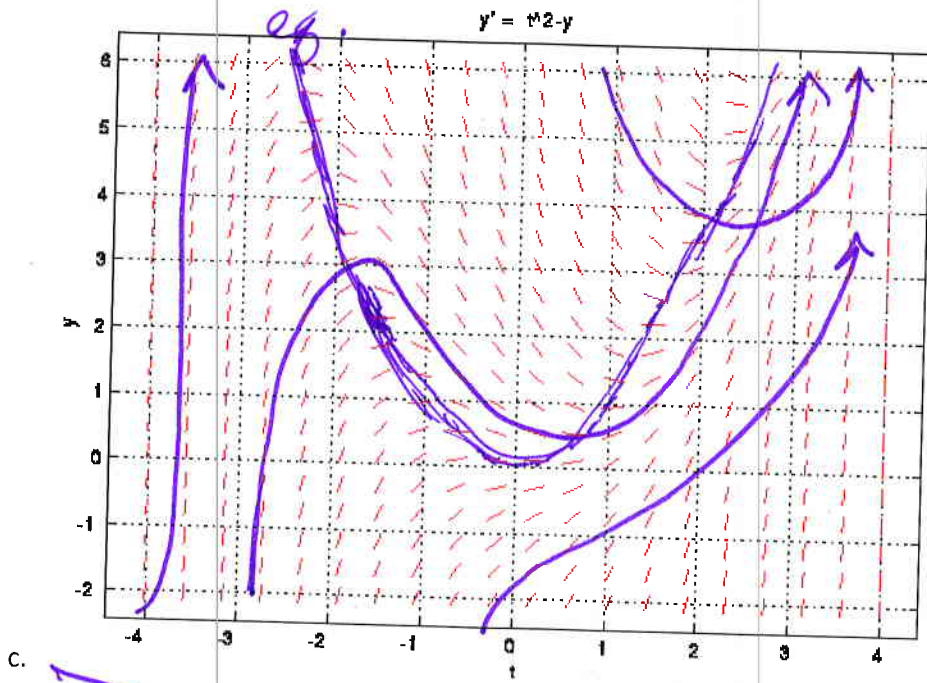
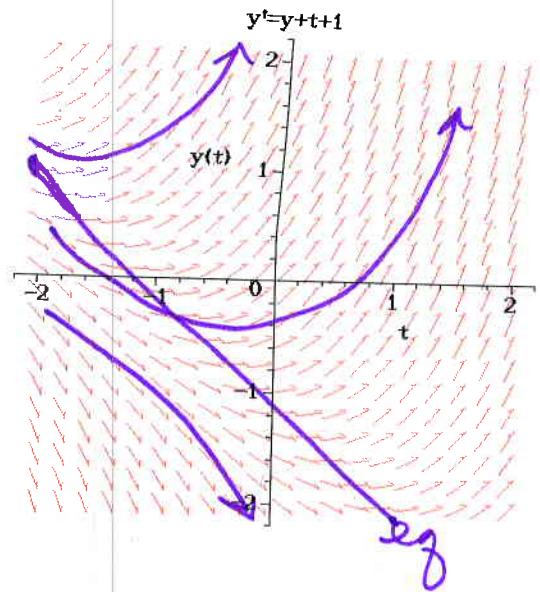
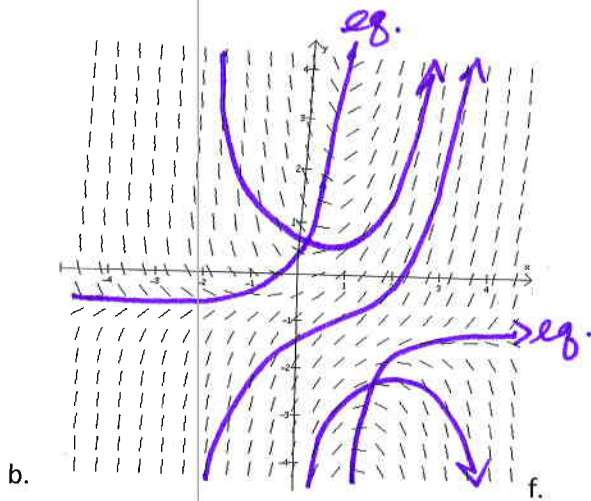
It is usually best to verify your analysis of a complicated direction field by graphing the whole field in the region of interest rather than just test points. For this purpose the direction field grapher at <http://www.dartmouth.edu/~rewn/dirfld.html> was used for this handout.



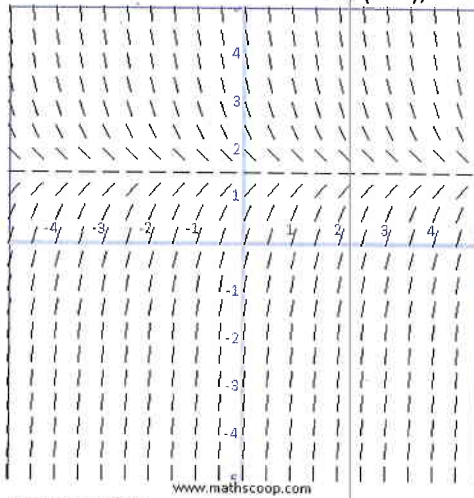
### Practice Problems.

- Below is a series of direction fields already graphed. Choose three different initial conditions and plot the solution curve going through that point both forwards and backwards in time. Whenever possible, note any equilibria, and state whether they are stable, unstable or semi-stable, or cannot be determined.



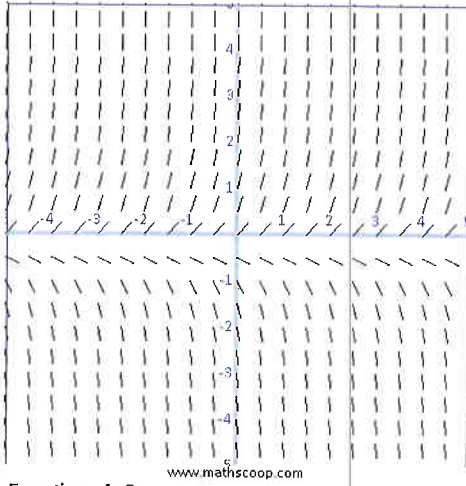
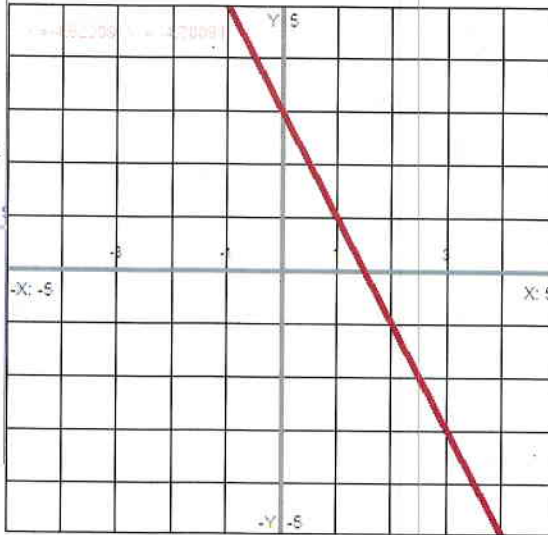


2. Plotted are the direction field (left), and phase plane (right).



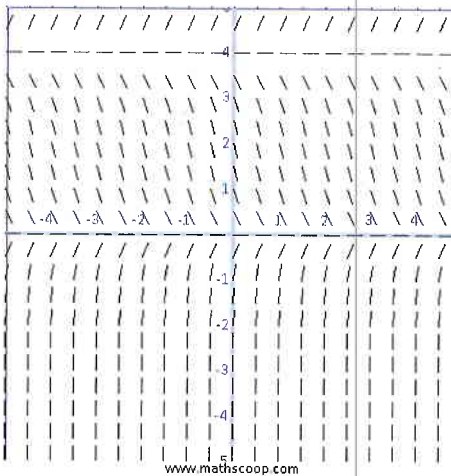
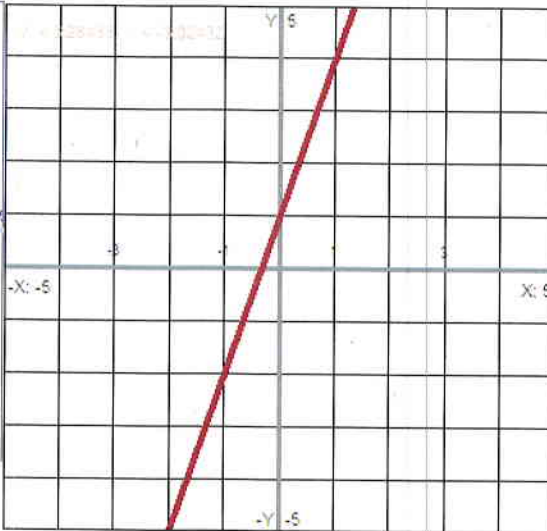
Equation :  $3-2y$

a.



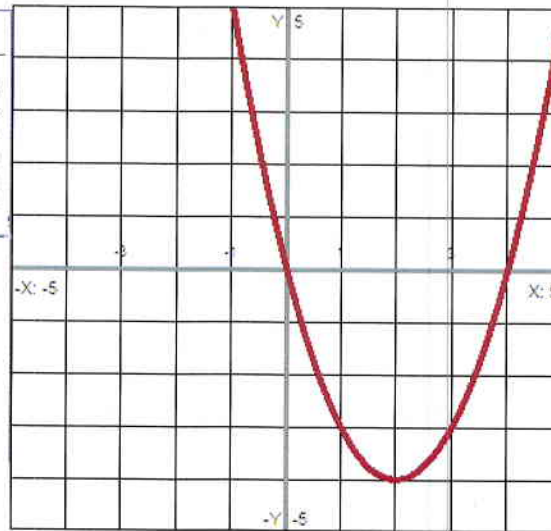
Equation :  $1+3y$

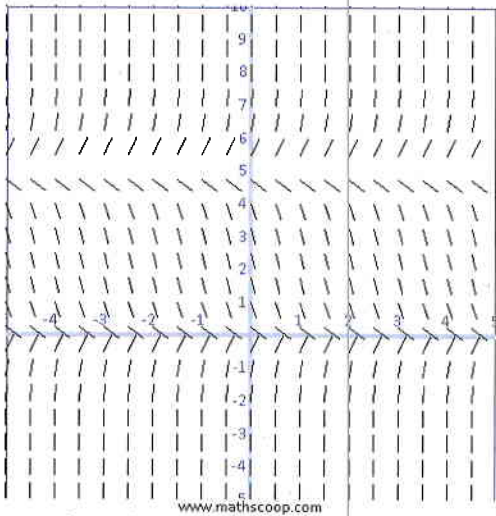
b.



Equation :  $y(y-4)$

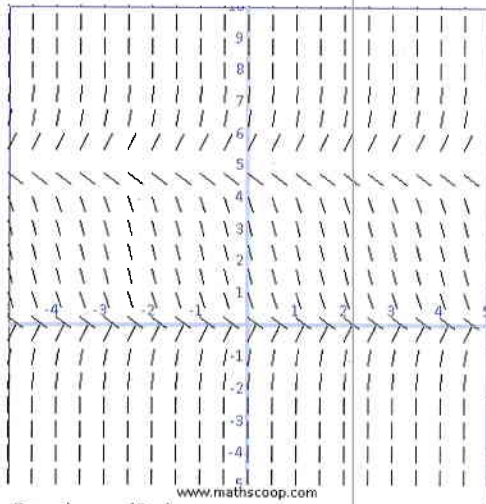
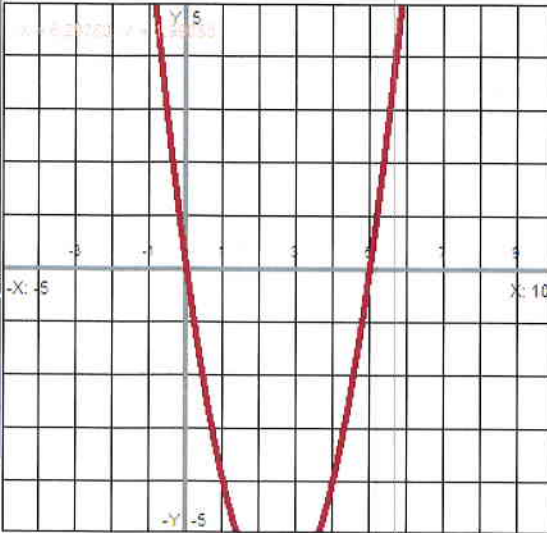
c.





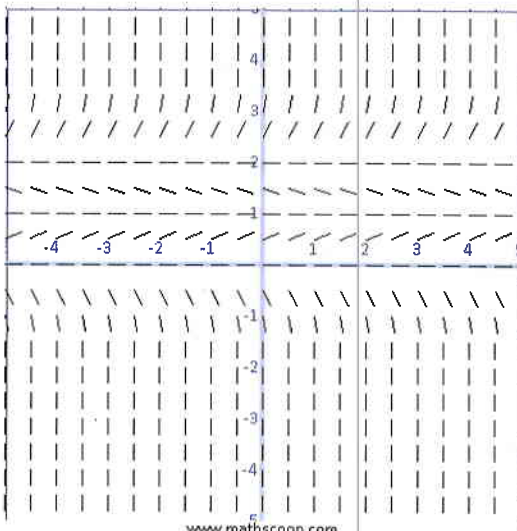
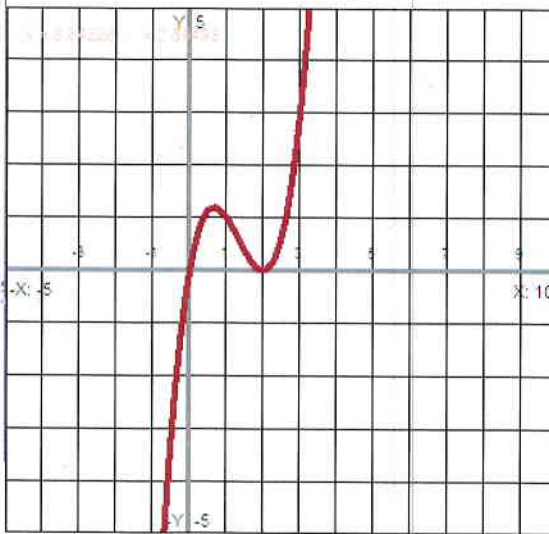
Equation :  $-y(5-y)$

d.



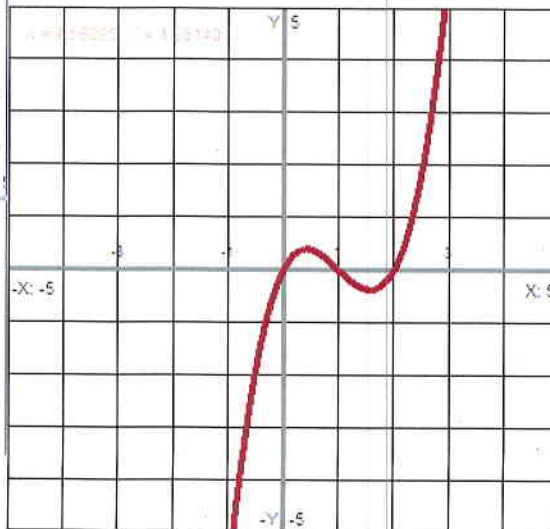
Equation :  $-y(5-y)$

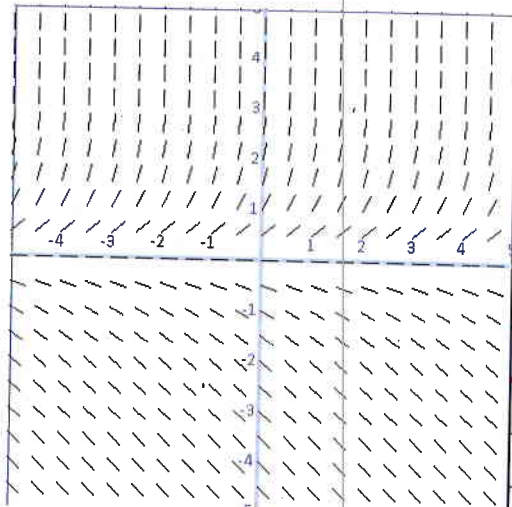
e.



Equation :  $y(y-1)(y-2)$

f.

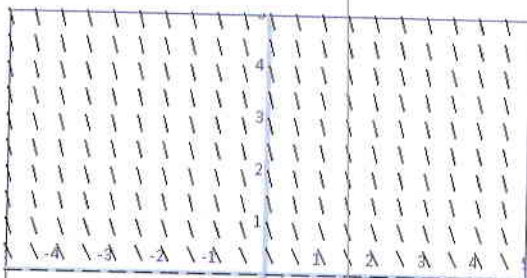
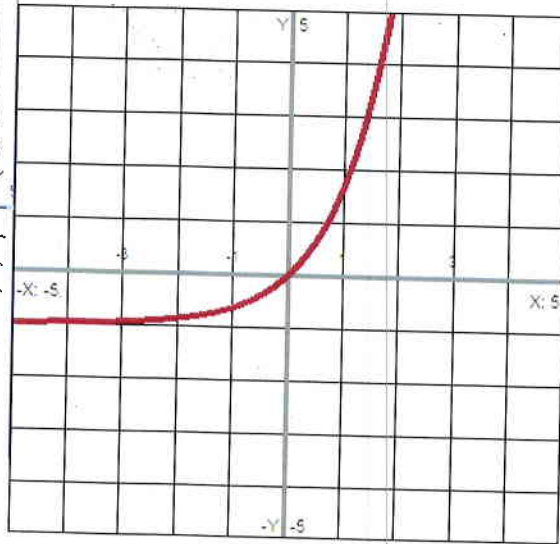




Equation :  $e^y - 1$

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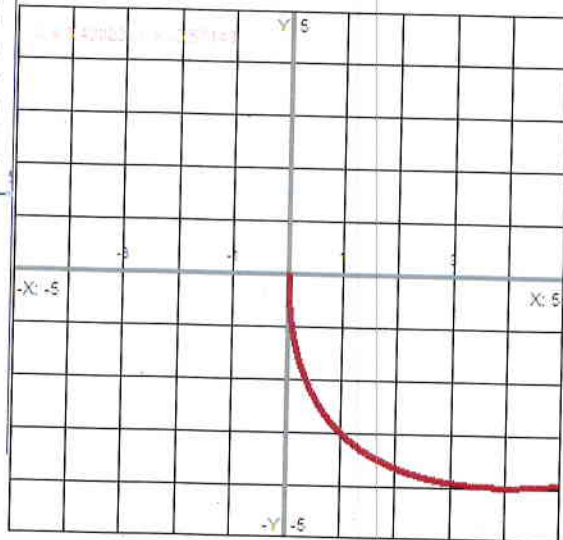
g.



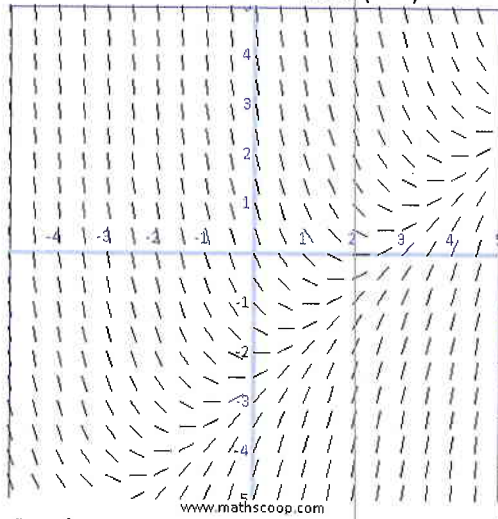
Equation :  $y - 4\sqrt{y}$

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h.

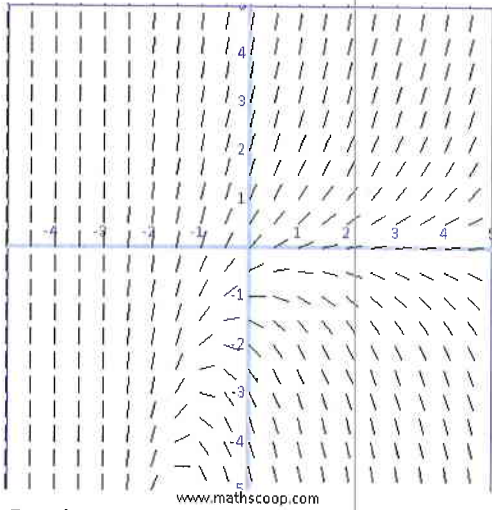
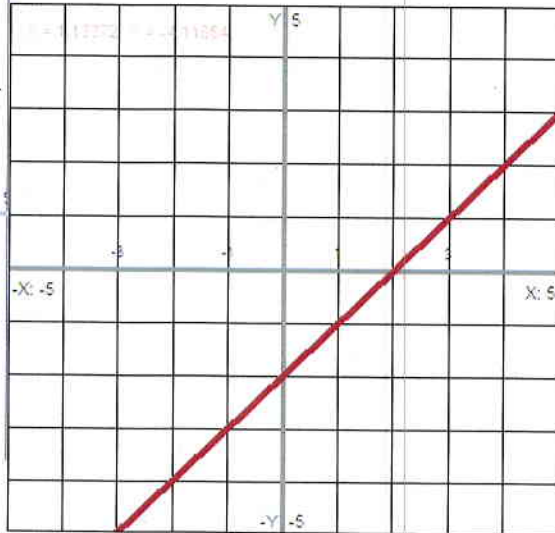


3. Plotted are the direction field (left) and the graph of the equilibrium solution (right).



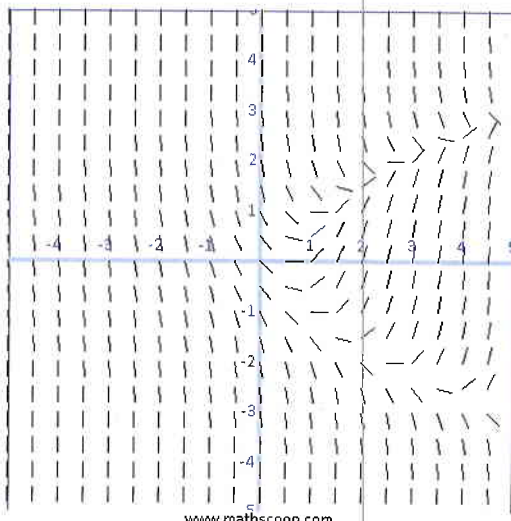
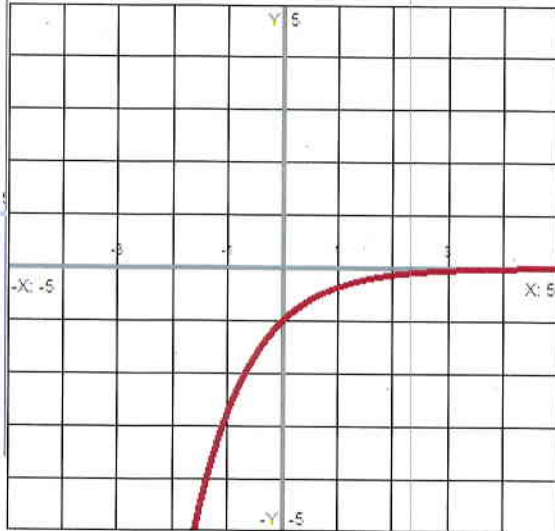
Equation :  $-2+x-y$

a.



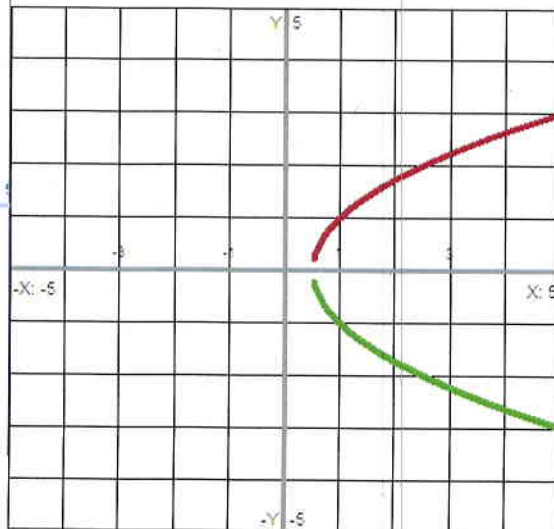
Equation :  $e^{-x}+y$

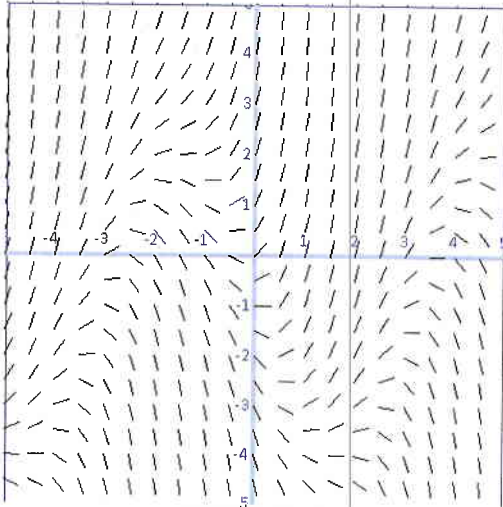
b.



Equation :  $2x-1-y^2$

c.

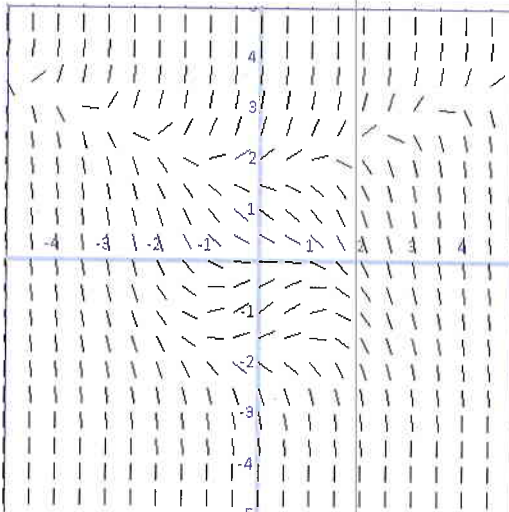
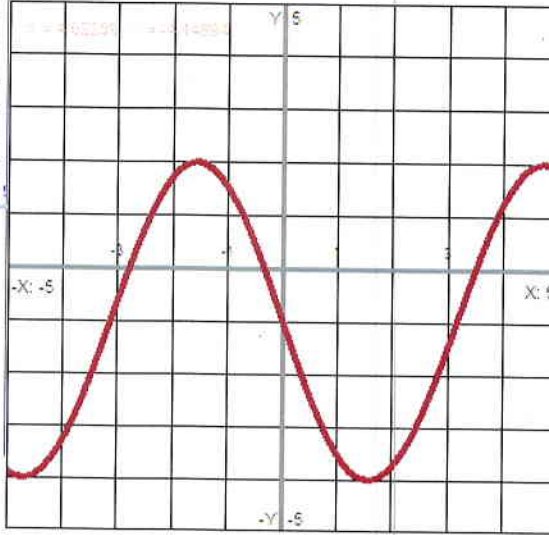




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Equation :  $3\sin(x)+1+y$

d.



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Equation :  $\frac{1}{3}y^3 - y - \frac{1}{2}x^2$

e.

