Stat 2470, Quiz #6, Spring 2014 Name KEY

Instructions: Show all work. You may use your calculator rather than compute formulas by hand, but if you do, 'show work' by saying which program you used to obtain the result and what information you entered. Round measures of center to one decimal place more than the data, and variance/standard deviation to two decimal places more than the original data. Round probabilities to three decimal places (or percent plus one decimal place).

1. A continuous random variable has a distribution given by $f(x) = \begin{cases} \frac{3}{4}x(x-2)^2, & 0 \le x \le 2 \\ 0, & otherwise \end{cases}$.

   Find the probability of obtaining a value for X between 1 and 1.5.

$$\int_1^{1.5} \frac{3}{4}x(x-2)^2\,dx = \frac{3}{4}\int_1^{1.5} x^3 - 4x^2 + 4x\,dx =$$

$$\frac{3}{4}\left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2\right]\Big|_1^{1.5} = \frac{3}{4}\left[\frac{1.5^4}{4} - \frac{4(1.5)^3}{3} + 2(1.5)^2\right] -$$

$$\frac{3}{4}\left[\frac{1}{4} - \frac{4}{3} + 2\right] = .2617$$

2. For the same distribution in Problem #1, find the mean and the variance of the distribution.

$$E(X) = \int_0^2 \frac{3}{4}x^2(x-2)^2\,dx = .8$$

$$V(X) = \int_0^2 \frac{3}{4}x^3(x-2)^2\,dx - (.8)^2 = .8 - .8^2 = .16$$