Stat 2470, 1/29 Discussion Questions Name ______________________________

Instructions: Attempt to answer these questions by reading the textbook or with online resources before coming to class on the date above.

1. What conditions are needed for using the hypergeometric distribution? What is the formula? State what each part of the formula refers to.

① population is finite (N)
② each event characterized as success or failure w/ M successes
③ sample of size n chosen w/o replacement

$$h(x; n, M, N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

N-M # of failures
n-x # of failures in sample w/ x successes

2. What is the expected value and variance of the hypergeometric distribution?

$$E(X) = n \cdot \frac{M}{N} \qquad V(X) = \left(\frac{N-n}{N-1}\right) \cdot n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right)$$

3. What is the finite population correction factor?

$$\left(\frac{N-n}{N-1}\right)$$

4. What conditions must be satisfied for the negative binomial distribution and how does it differ from the binomial distribution?

① sequence of independent trials
② each trial either success or failure
③ probability of success constant from trial to trial
④ experiment continues until a fixed # of successes (r) is achieved

5. What is the formula for the negative binomial distribution? Label any key features.

$$nb(x; r, p) = \binom{x+r-1}{r-1} p^r (1-p)^x \qquad x = 0, 1, 2, \ldots$$

x is # of failures before rth success
(-1) happens because last term must be a success so only previous x+r-1 terms can be reordered.

6. What is the expected value and variance of the negative binomial distribution?

$$E(x) = \frac{r(1-p)}{p} \qquad V(x) = \frac{r(1-p)}{p^2}$$

7. How is the geometric distribution related to the negative binomial distribution?

the geometric distribution is a special case of the negative binomial w/ $r=1$

8. What is the formula for the Poisson distribution?

$$p(x;\mu) = \frac{e^{-\mu}\mu^x}{x!} \quad x = 0, 1, 2, 3, \ldots$$

9. What kind of events does Poisson model? Give at least two examples.

# of events that occur in a fixed time frame. Cars driving up to drive through window; # of motorcycle accidents per year; # of atoms that decay in a second

10. How are the binomial and Poisson related?

if we let # of trials go to infinity and the probability go to zero so that $np \to$ # not zero, then $b(x;n,p) \to p(x;\mu)$ w/ $\mu = np$

11. What is the expected value and the variance of the Poisson distribution?

$E(X) = \mu \qquad V(X) = \mu$

12. What is a Poisson process and how can we use this fact to adjust our Poisson distribution for different time frames?

① There is a parameter $\alpha > 0$ such that for any short time interval of length Δt the probability that exactly one event occurs is $\alpha \cdot \Delta t + o(\Delta t)$ [$o(\Delta t) \to 0$ faster than Δt]

② The probability of more than one event occurring during Δt is $o(\Delta t)$

③ The number of events occurring the time interval Δt is independent of the # that occurs prior to Δt.

$\Rightarrow \mu = \alpha t$ as we adjust times, we scale the mean proportionately