Stat 2470, 2/19 Discussion Questions Name ______________________

Instructions: Attempt to answer these questions by reading the textbook or with online resources before coming to class on the date above.

1. Give at least two examples (one discrete and one continuous) of how these general concepts of joint probability distributions can be extended to three or more variables.

multinomial functions

or in the general continuous case $f(x,y,z)$ joint pdf for 3 variables w/ condition that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)\,dz\,dy\,dx = 1$

2. Give the formula for the conditional probability density function.

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}$$

3. Explain the steps to calculate this formula to find $f_{X|Y}(x|y)$. Then give the conditional formula for one specific distribution (your choice) and one specific value of y (your choice).

switch roles

find $f_X(x)$ first.

$p(x,y)$		y: 0	1	2
x	0	.10	.04	.02
	1	.08	.20	.06
	2	.06	.14	.30

$f_X(x) =$

x	0	1	2
$f_X(x)$	.16	.34	.5

for $x = 0$

$f_{y|x}$ for $x=0$

y	0	1	2
$f_{y\|x}$	$\frac{.10}{.16}$	$\frac{.04}{.16}$	$\frac{.02}{.16}$

4. How do the discrete and continuous procedures differ above?

when dealing w/ discrete variables you get a table of values
when working w/ continuous variables, you get a function

5. How do we calculate expected values of X? $E(X)$? How do the discrete and continuous cases differ?

$E(x)$ for discrete is $\sum_x \sum_y x \cdot p(x,y)$

$E(X)$ for continuous is $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f(x,y)\,dy\,dx$

6. What about $E(Y)$? $E(XY)$?

$E(y) = \sum_x \sum_y y \cdot p(x,y)$ or $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y\, f(x,y)\,dy\,dx$

$E(xy) = \sum_x \sum_y xy\, p(x,y)$ or $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\,dy\,dx$

7. How if we wanted $E(|X-Y|)$? Explain the steps. we need to split the integral where $y=x$ since the sign of $x-y$ changes there

$$\left|\int_{-\infty}^{\infty}\int_{-\infty}^{x}(x-y)f(x,y)\,dy\,dx\right| + \left|\int_{-\infty}^{\infty}\int_{x}^{\infty}(x-y)f(x,y)\,dy\,dx\right|$$ in continuous case

discrete case is similar but w/ sums when $x<y$ & sums $x \geq y$

8. What is the formula for the covariance of X and Y? $Cov(X,Y)$? How do the discrete and continuous cases differ?

$$Cov(X,Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\mu_x)(y-\mu_y)\,f(x,y)\,dy\,dx$$ or

$$\sum_x\sum_y (x-\mu_x)(y-\mu_y)\,p(x,y)$$

Same except for change between sums & integrals

9. What is the alternate version for the formula for covariance?

$$Cov(X,Y) = E(XY) - \mu_X\cdot\mu_Y \quad \text{or} \quad E(XY) - E(X)E(Y)$$

10. Express the alternate version in summations and/or integrals.

Continuous $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\,f(x,y)\,dy\,dx - \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y)\,dy\,dx\right]\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f(x,y)\,dy\,dx\right]$$

11. What does it mean if the covariance is zero? Or if $E(XY) = E(X)E(Y)$?

it could mean that they are independent, but generally only that there is no linear relationship

12. How do we compute the correlation coefficient $Corr(X,Y) = \rho_{XY}$?

$$\frac{Cov(X,Y)}{\sigma_X\cdot\sigma_Y}$$

σ_X recall $= \sqrt{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f(x,y)\,dy\,dx - \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y)\,dy\,dx\right]^2}$

similarly for σ_Y

13. The formula above needs σ_X and σ_Y. How do we calculate those values? Put another way, how do we find $V(X)$ and $V(Y)$?

$$V(X) = E(X^2) - [E(X)]^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f(x,y)\,dy\,dx - \left[\int_{-\infty}^{\infty} x f(x,y)\,dy\,dx\right]^2$$

in continuous case

in discrete case

$$V(Y) = E(Y^2) - [E(Y)]^2 = \sum_x \sum_y y^2 p(x,y) - \left[\sum_x \sum_y y\,p(x,y)\right]^2$$

14. How is the correlation affected by a linear transformation of the variables X and Y?

it's not

$U = A + BX$
$V = C + DY$

$\rho_{XY} = \rho_{UV}$

15. What is the range of values ρ_{XY} can take?

$$-1 \leq \rho_{XY} \leq 1$$

16. What does $\rho = \pm 1$ mean? What does $\rho = 0$ mean?

no variability. perfect linear correlation

no linear correlation

17. If we know $\rho = 0$, can we assert that X and Y are independent? Why or why not?

no. they may be strongly dependent but nonlinear relationship