Stat 2470, Exam #2, Spring 2015 Name KEY

Instructions: Show all work. State any formulas used. If you use the calculator, you should say which function you used, and what you entered into it, as well as any output. I can only give partial correct for incorrect answers if I have something to grade.

1. Find the value of k that will make $f(x,y) = \begin{cases} ky\sqrt[4]{x}, & 0 \le x \le 4, 0 \le y \le 2 \\ 0, & otherwise \end{cases}$ a legitimate probability distribution. (10 points)

$$k\int_0^4\int_0^2 y x^{1/4}\,dy\,dx = \frac{k}{2}\int_0^4 y^2 x^{1/4}\Big|_0^2 dx = 2k\int_0^4 x^{1/4}dx =$$

$$\frac{4\cdot 2}{5}k\,x^{5/4}\Big|_0^4 = \frac{8}{5}k\,(4)^{5/4} = \frac{8}{5}k\,(2)^{5/2} = \frac{8}{5}k\cdot 4\sqrt{2} = 1 \Rightarrow \frac{32\sqrt{2}k}{5} = 1$$

$$k = \frac{5}{32\sqrt{2}}$$

2. Use the distribution in #1, to find the marginal distribution $f_Y(y)$. (6 points)

$$\frac{5}{32\sqrt{2}}\int_0^4 y\sqrt[4]{x}\,dx = \frac{5}{32\sqrt{2}}y\int_0^4 x^{1/4}dx = \frac{5}{\cancel{32}_{8}\sqrt{2}}y\cdot\frac{\cancel{4}}{\cancel{5}}x^{5/4}\Big|_0^4 =$$

$$\frac{1}{8\sqrt{2}}y\,(4)^{5/4} = \frac{1}{8\sqrt{2}}y\,(2)^{5/2} = \frac{1}{\cancel{8}_{2}\cancel{\sqrt{2}}}y\cdot\cancel{4}\cancel{\sqrt{2}} = \frac{1}{2}y = f(y)$$

3. Use the distribution in #1, and the value of k you found, to find $E(Y)$ of the distribution. (7 points)

$$E(Y) = \frac{5}{32\sqrt{2}}\int_0^4\int_0^2 y^2x^{1/4}\,dy\,dx = \frac{5}{32\sqrt{2}}\int_0^4 \frac{y^3}{3}x^{1/4}\Big|_0^2 dx = \frac{5}{\cancel{32}_{4}\sqrt{2}}\int_0^4 \frac{\cancel{8}}{3}x^{1/4}dx$$

$$= \frac{5}{12\sqrt{2}}\int_0^4 x^{1/4}dx = \frac{\cancel{5}}{\cancel{12}_{3}\sqrt{2}}\cdot\frac{\cancel{4}}{\cancel{5}}x^{5/4}\Big|_0^4 = \frac{\cancel{5}}{\cancel{12}_{3}\cancel{\sqrt{2}}}\cdot\frac{\cancel{4}}{\cancel{5}}\cdot 4\cancel{\sqrt{2}} = \frac{4}{3}$$

4. Use the probability distribution in #1, to find $P(0 \le X \le 1, and\ 1 \le Y \le 2)$. (7 points)

$$\frac{5}{32\sqrt{2}}\int_0^1\int_1^2 y x^{1/4}\,dy\,dx = \frac{5}{32\sqrt{2}}\int_0^1 \frac{y^2}{2}\Big|_1^2 x^{1/4}\,dx = \frac{5}{32\sqrt{2}}\int_0^1 (2-\tfrac{1}{2})x^{1/4}\,dx$$

$$= \frac{15}{64\sqrt{2}}\int_0^1 x^{1/4}\,dx = \frac{\cancel{15}^{3}}{\cancel{64}_{16}\sqrt{2}}\cdot\frac{\cancel{4}}{\cancel{5}}x^{5/4}\Big|_0^1 = \frac{1}{16\sqrt{2}} \approx .04419$$

5. A discrete joint probability mass function is shown in the table below.

$f(x,y)$	y = 0	1	2	3	4
x = 0	0.03	0.06	0.08	0.1	0.13
1	0.12	0.08	0.05	0.05	0.04
2	0.05	0.05	0.06	0.06	0.04

a. Find the marginal distribution function $f_Y(y)$. (6 points)

y	0	1	2	3	4
p(y)	.2	.19	.19	.21	.21

b. Find $E(XY)$. (10 points)

$E(xy) = 0\cdot0(.03) + 0\cdot1(.06) + 0\cdot2(.08) + 0(3)(.10) + 0\cdot4(.13) +$
$1(0)(.12) + 1\cdot1(.08) + 1\cdot2(.05) + 1\cdot3(.05) + 1\cdot4(.04)$
$+ 2(0)(.05) + 2\cdot1(.05) + 2\cdot2(.06) + 2\cdot3(.06) + 2\cdot4(.04) =$

1.51

c. Find $f_{X|Y}(x|y)$ for $Y = 1$. (6 points)

$\frac{f(x,y)}{f(y)}$

X	0	1	2
$f_{X\|Y}$	$\frac{.06}{.19} = \frac{6}{19}$	$\frac{.08}{.19} = \frac{8}{19}$	$\frac{.05}{.19} = \frac{5}{19}$
	$\approx .316$	$\approx .421$	$\approx .263$

6. Give an example of a parameter (or feature of a distribution) with multiple unbiased estimators. Give at least two of those unbiased estimators. (5 points)

measure of center can be done w/ mean, median or trimmed mean. all work pretty well, but mean has smallest variance

answers may vary

7. Consider the data in the table below. Suppose this data is distributed as a Poisson distribution. Find the maximum likelihood function and use it to find an estimate for μ. (12 points)

46	**15**	**24**	**13**	**5**
18	11	10	3	8

$$L(\mu) = \frac{e^{-\mu}\mu^{46}}{46!}\cdot\frac{e^{-\mu}\mu^{15}}{15!}\cdot\frac{e^{-\mu}\mu^{24}}{24!}\cdot\frac{e^{-\mu}\mu^{13}}{13!}\cdot\frac{e^{-\mu}\mu^{5}}{5!}\cdot\frac{e^{-\mu}\mu^{18}}{18!}\cdot$$

$$\frac{e^{-\mu}\mu^{4}}{11!}\cdot\frac{e^{-\mu}\mu^{10}}{10!}\cdot\frac{e^{-\mu}\mu^{3}}{3!}\cdot\frac{e^{-\mu}\mu^{8}}{8!}$$

← neglect denominator does not affect result. Call it K

$$L(\mu) = \frac{e^{-10\mu}\mu^{153}}{K} \qquad \frac{\partial L}{\partial \mu} = \frac{1}{K}\left(-10e^{-10\mu}\mu^{153} + e^{-10\mu}\cdot 153\mu^{152}\right) = 0$$

$$\frac{1}{K}e^{-10\mu}\mu^{152}\left(-10\mu + 153\right) = 0 \Rightarrow 153 = 10\mu \Rightarrow \mu = \frac{153}{10} = \boxed{15.3}$$

8. A particular state has a law that beer must have a lower alcohol content than 6% to be considered legally beer and not wine. A brewer measures the alcohol content of ten bottles of beer and finds the following results.

6.7	**5.7**	**5.6**	**5.3**	**5.9**	**5.8**
6.1	6.1	5.7	6.4	5.9	5.5

a. Find the sample mean and standard deviation. (5 points)

1 Var Stats

$\bar{X} = 5.89$

$S = .3895$

b. Find a 99% confidence interval for the data. Does it contain any values that are greater than 6%? (7 points)

TInterval Data

List: L1

Freq: 1

C-level: .99

(5.5424, 6.2409)

it does contain values greater than 6%

c. If the brewer has registered his beer with the state as being 5.6% by volume, conduct a hypothesis test to determine if this sample provides sufficient evidence to believe that the alcohol content is higher than what he thinks it should be at the 5% significance level. Clearly state the null and alternative hypotheses, the test statistic, and p-value. Then interpret the results of the test in the context of the problem. (12 points)

$H_0: \mu = 5.6$

$H_a: \mu > 5.6$

T-Test Data

$\mu_0 = 5.6$

List: L1 Freq: 1

$\mu > \mu_0$

$\Rightarrow t = 2.59$

$p = .0124859$ — this is less than 5% so we conclude Ho is rejected. The brewer does have too high of alcohol content

d. How can we use the confidence interval we found in part (b) to conduct the test in part (c) at the 1% significance level? (5 points)

Since the 99% confidence interval contains 5.6, it means that at the 1% level there is not enough evidence to reject Ho, which agrees w/ the p-value > .01

9. Suppose that Gallup wishes to conduct a Presidential poll to determine who in the front-runner in an upcoming election is. They ask 2103 people whether they prefer the Democrat or the Republican and they find that 51% of respondents chose the Republican candidate. Conduct a hypothesis test on these results to determine if this is sufficiently strong evidence to support the claim that more Americans support the Republican candidate than the Democratic one? Clearly state your null and alternative hypotheses, and interpret the results in the context of the problem. (12 points)

1 PropZTest

$H_o: p = .5$

$H_a: p > .5$

$X = .51 * 2103 = 1072.53 \Rightarrow 1073$

$n = 2103$

$\Rightarrow$ $z = .93766$

$p = .1742$

fail to reject Ho

there is not enough evidence to conclude that the public really favors Republicans more than 50% of the time.

10. What conditions must be satisfied to use a Z-test instead of a T-test when conducting hypothesis testing (or constructing a confidence interval)? (5 points)

σ must be known, distribution normal

and $n > 40$

11. Suppose that you wish to conduct a hypothesis test: your null hypothesis asserts that the true mean of the population is 500 mL, but your preliminary sample suggests the mean is 510 mL. What sample size is needed under these circumstance with a 1% significance level, and wish the chance of a Type II error to be less than 5%. (8 points)

$\sigma = 15 mL$

$H_o: \mu = 500$

$H_a: \mu > 500$

$Z_\alpha = 2.326$

$Z_\beta = 1.645$

$$n = \left[\frac{15(2.326+1.645)}{500-510}\right]^2$$

$$n = [5.9565]^2 = 35.47989$$

$\boxed{n = 36}$

12. Convert the following test statistics to P-values. Say whether you'd reject or fail to reject the hypothesis under the stated conditions at the given level of significance. (5 points each)

a. $z = 1.13, \alpha = 0.05$, one-tailed

$P = .129$ fail to reject

b. $z = -8.59, \alpha = 0.01$, two-tailed

$P = 8.808 \times 10^{-18}$ reject

c. $t = 6.52, df = 11, \alpha = 0.001$, one-tailed

$P = 2.15 \times 10^{-5}$ reject

d. $t = 0.799, n = 5, \alpha = 0.10$, two-tailed

$P = .469$ fail to reject