Stat 2470, Quiz #5, Spring 2015 Name KEY

Instructions: Show all work. State any formulas used. If you use the calculator, you should say which function you used, and what you entered into it, as well as any output. I can only give partial correct for incorrect answers if I have something to grade.

1. Consider the distribution given by $f(x) = \begin{cases} k\ln(x), & 1 \le x \le e \\ 0, & otherwise \end{cases}$. Find the value of k that makes this a legitimate probability distribution. Then find the equation for the cumulative distribution.

$k\int_1^e \ln x\,dx$ $u=\ln x$ $dv=dx$ $du=\frac{1}{x}dx$ $v=x$

$x\ln x - \int_1^e \frac{1}{x}\cdot x\,dx =$

$x\ln x - \int_1^e dx =$

$x\ln x - \cancel{x}\Big|_1^e =$

$e(1) - e - [(1)(0) - 1] = 1$ $k=1$

$\int_1^x \ln x\,dx = x\ln x - x\Big|_1^x$

$x\ln x - x + 1 = F(x)$

2. Use the function above, and the value of k you found, to find the expected value and the variance of the distribution.

$\int_1^e x\ln x\,dx$ $u=\ln x$ $dv=x\,dx$ $du=\frac{1}{x}dx$ $v=\frac{1}{2}x^2$

$\frac{1}{2}x^2\ln x - \int_1^e \frac{1}{2x}x^2\,dx =$

$\frac{1}{2}x^2\ln x - \frac{1}{2}\int_1^e x\,dx =$

$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\Big|_1^e =$

$\frac{1}{2}e^2(1) - \frac{1}{4}e^2 - 0 + \frac{1}{4}$

$\boxed{\frac{1}{4}e^2 + \frac{1}{4}} = E(X)$

$E(X^2)$

$\int_1^e x^2\ln x\,dx$ $u=\ln x$ $dv=x^2dx$ $du=\frac{1}{x}dx$ $v=\frac{1}{3}x^3$

$\frac{1}{3}x^3\ln x - \int_1^e \frac{1}{3}x^3\cdot\frac{1}{x}dx =$

$\frac{1}{3}x^3\ln x - \frac{1}{3}\int_1^e x^2\,dx =$

$\frac{1}{3}x^3\ln x - \frac{1}{9}x^3\Big|_1^e =$

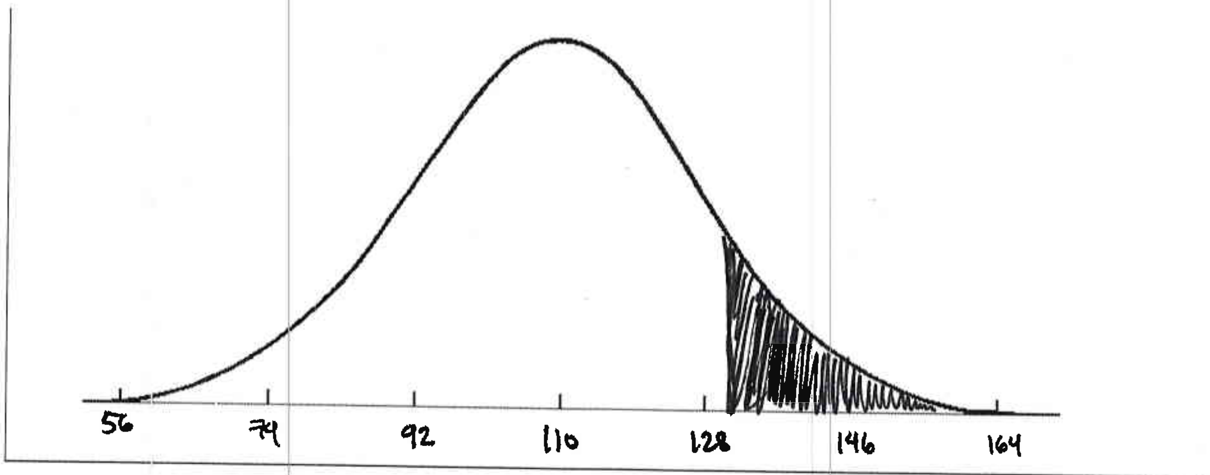
$\frac{1}{3}e^3(1) - \frac{1}{9}e^3 - \frac{1}{3}(1)(0) + \frac{1}{9}(1)$

$\frac{2}{9}e^3 + \frac{1}{9}$

$V(X) = E(X^2) - [E(X)]^2 = \frac{2}{9}e^3 + \frac{1}{9} - \left(\frac{1}{4}e^2 + \frac{1}{4}\right)^2$

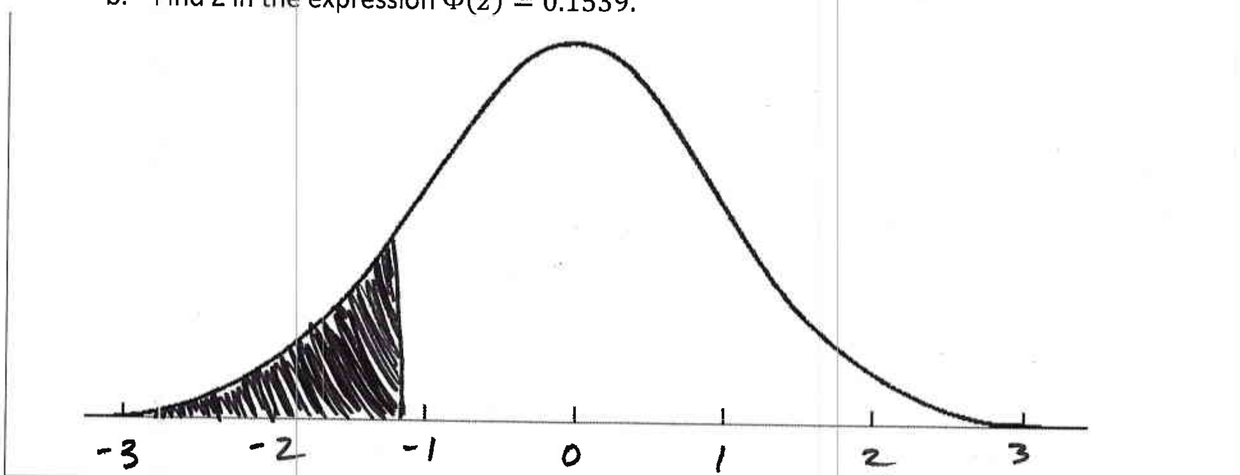
3. Plot the information on the graph of the normal distributions below. Find the x (or z) value where the percent under the curve is given, or find the percentage under the curve where the x (or z) value is given.

a. Mean is 110, the standard deviation is 18. Find $P(X > 130)$.



normalcdf(130, E99, 110, 18) = .133 or 13.3%

b. Find Z in the expression $\Phi(z) = 0.1539$.



$Z \approx -1.01984$

4. The gamma distribution is given by $f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}, x \geq 0 \\ 0, \quad otherwise \end{cases}$. Find the probabilities associated with the information given below.

$\Gamma(\alpha) = \Gamma(2) = 1$

a. $P(0 \leq X \leq 1), \alpha = 2, \beta = 3$

$\int_0^1 \frac{1}{3^2(1)} x e^{-x/3} dx = .0446249$

b. $P(X \geq 4), \alpha = 7, \beta = 2$

$\int_4^{\infty} \frac{1}{2^7(720)} x^6 e^{-x/2} dx = 1 - \int_0^4 \frac{1}{128 \cdot 720} x^6 e^{-x/2} dx =$ $\quad \Gamma(7) = 6!$

$.995466$