Stat 2470, Quiz #6, Spring 2015 Name KEY

Instructions: Show all work. State any formulas used. If you use the calculator, you should say which function you used, and what you entered into it, as well as any output. I can only give partial correct for incorrect answers if I have something to grade.

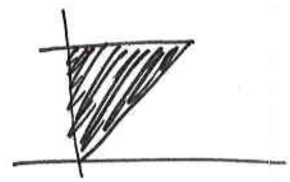
1. Find the value of k that makes $f(x,y) = \begin{cases} kx^2y^4, 0 \le x \le 1, x \le y \le 1 \\ 0, \quad\quad\quad\quad otherwise \end{cases}$ a legitimate probability distribution.

$$\int_0^1\int_x^1 kx^2y^4\,dy\,dx = k\int_0^1 x^2\frac{y^5}{5}\Big|_x^1 dy = \frac{k}{5}\int_0^1 x^2(1-x^5)\,dx =$$

$$\frac{k}{5}\int_0^1 x^2 - x^7\,dx = \frac{k}{5}\left[\frac{x^3}{3}-\frac{x^8}{8}\right]_0^1 = \frac{k}{5}\left[\frac{1}{3}-\frac{1}{8}\right] = \frac{k}{\cancel{5}}\left[\frac{\cancel{5}}{24}\right] = 1$$

$k = 24$

$$f(x,y) = \begin{cases} 24x^2y^4 & 0 \le x \le 1,\ x \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$



2. Use the information in #1 to find $f_X(x)$.

$$24\int_x^1 x^2y^4\,dy = 24x^2\frac{y^5}{5}\Big|_x^1$$

$$= 24x^2\left[\frac{1}{5}-\frac{x^5}{5}\right] = \frac{24}{5}(x^2 - x^7)$$

3. A discrete joint probability mass function is shown in the table below.

$f(x,y)$		y = 0	1	2	3	4	5
x	0	0.02	0.05	0.07	0.11	0.14	0.19
	1	0.18	0.09	0.06	0.04	0.03	0.02

a. Find $P(X = 1, 2 \le Y \le 4)$

$.06 + .04 + .03 = .13$

b. Find the marginal distribution function $f_Y(y)$.

Y	0	1	2	3	4	5
p(y)	.2	.14	.13	.15	.17	.21