Stat 2470, Quiz #8, Spring 2015 Name KEY

Instructions: Show all work. State any formulas used. If you use the calculator, you should say which function you used, and what you entered into it, as well as any output. I can only give partial correct for incorrect answers if I have something to grade.

1. Consider the data in the table below.

27	**12**	**5**	**36**	**1**
14	6.2	16.3	2	12

a. Suppose this data is distributed according to a gamma distribution with $\mu = \alpha\beta$ and $\sigma^2 = \alpha\beta^2$. Use the method of moments to find the parameters α and β.

$\frac{1}{n}\sum x_i = \frac{131.5}{10} = 13.15 = \alpha\beta = E(X)$

$\frac{1}{n}\sum x_i^2 = \frac{2843.13}{10} = 284.313 = E(X^2)$

$\sigma^2 = V(X) = E(X^2) - [E(X)]^2$

$284.313 - (13.15)^2 = 111.3905 = \alpha\beta^2$

$\alpha = \frac{13.15}{\beta}$ $\quad 111.3905 = \left(\frac{13.15}{\beta}\right)\beta^2 \Rightarrow 111.3905 = 13.15\beta \Rightarrow \beta \approx 8.47$

$\alpha = \frac{13.15}{8.47} \approx 1.55$

$\boxed{\alpha = 1.55,\ \beta = 8.47}$

b. Suppose this data is distributed as an exponential distribution. Find the maximum likelihood function and use it to find an estimate for λ. [Hint: the exponential distribution is $f(x) = \lambda e^{-\lambda x}, x \geq 0$.]

$L(\lambda) = \lambda e^{-27\lambda} \cdot \lambda e^{-12\lambda} \cdot \lambda e^{-5\lambda} \cdot \lambda e^{-36\lambda} \cdot \lambda e^{-\lambda} \cdot \lambda e^{-14\lambda} \cdot \lambda e^{-6.2\lambda} \cdot \lambda e^{-16.3\lambda}$

$\cdot \lambda e^{-2\lambda} \cdot \lambda e^{-12\lambda} = \lambda^{10} e^{-131.5\lambda}$

$\frac{\partial L}{\partial \lambda} = 10\lambda^9 e^{-131.5\lambda} - 131.5\lambda^{10} e^{-131.5\lambda} = \lambda^9 e^{-131.5\lambda}[10 - 131.5\lambda] = 0$

$10 - 131.5\lambda = 0$

$10 = 131.5\lambda$

$\lambda = \frac{10}{131.5} \approx .076$