Math 2568, Exam #1, Part I, Spring 2015 Name KEY

Instructions: Show all work. You may **not** use a calculator on this portion of the exam. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals). Reduce as much as possible. Be sure to complete all parts of each question. Provide explanations where requested. When you are finished with this portion of exam, get Part II.

1. Write the system of equations $\begin{cases} 2x_1 + 5x_2 = 7 \\ x_1 - 2x_2 = 8 \end{cases}$ as a) a vector equation, b) a matrix equation, c) an augmented matrix. (6 points)

a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 5 \\ -2 \end{bmatrix} x_2 = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

c) $\left[\begin{array}{cc|c} 2 & 5 & 7 \\ 1 & -2 & 8 \end{array}\right]$

2. Row reduce the system to obtain the solution $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. (6 points)

$R_2 \leftrightarrow R_1$

$\left[\begin{array}{cc|c} 1 & -2 & 8 \\ 2 & 5 & 7 \end{array}\right]$

$-2R_1 + R_2 \rightarrow R_2$

$-2 \quad 4 \quad -16$

$\left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 9 & -9 \end{array}\right]$

$\frac{1}{9} R_2 \rightarrow R_2$

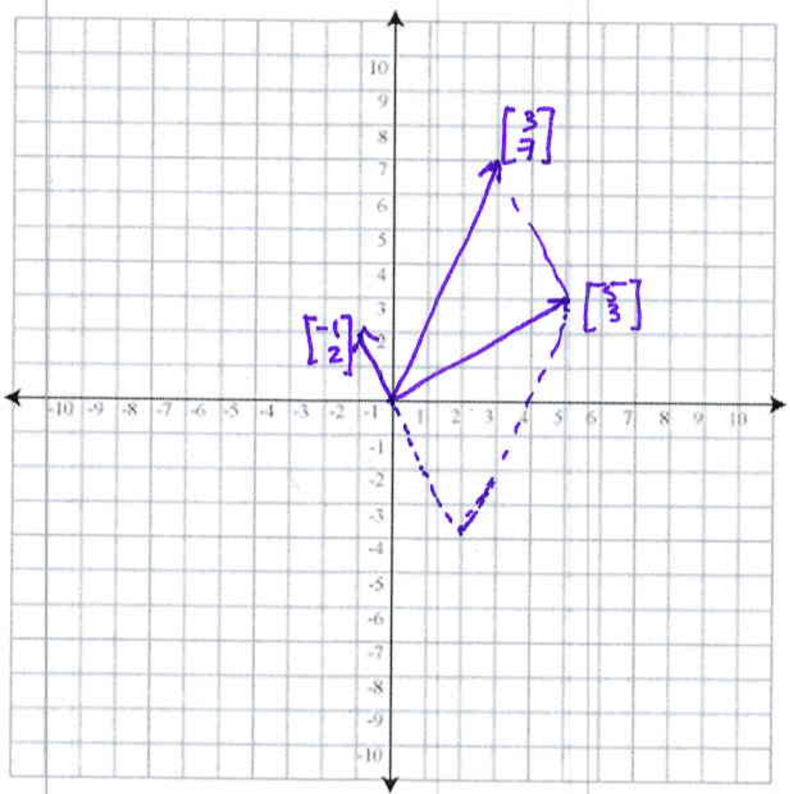
$\left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 1 & -1 \end{array}\right]$

$2R_2 + R_1 \rightarrow R_1$

$\left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & -1 \end{array}\right]$

$\vec{x} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$

3. The solution to the system $x_1\begin{bmatrix}-1\\2\end{bmatrix} + x_2\begin{bmatrix}3\\7\end{bmatrix} = \begin{bmatrix}5\\3\end{bmatrix}$ is $\vec{x} = \begin{bmatrix}-2\\1\end{bmatrix}$. Represent the solution graphically on the graph below. (8 points)



4. Determine if each statement is True or False. (2 points each)

a. T F Two matrices are row equivalent if they have the same dimensions.

b. T F Two fundamental questions about linear systems is about existence and uniqueness.

c. T F Both $\begin{bmatrix}1 & * & * & *\\0 & 1 & * & *\\0 & 0 & 0 & 1\end{bmatrix}$ and $\begin{bmatrix}0 & 1 & * & *\\0 & 0 & * & *\\0 & 0 & 1 & 0\end{bmatrix}$ are matrices in echelon form.

d. T F The reduced echelon form of a matrix is always unique.

e. T F If two points corresponding to two vectors line on the same line, then the vectors they represent are linearly dependent.

f. T F The $span\{\vec{u}, \vec{v}\}$ is just the lines passing through the point $\vec{u}$ and the origin, and the line passing through the point $\vec{v}$ and the origin.

g. T F The equation $A\vec{x} = \vec{b}$ is consistent if the augmented matrix representing the system has a pivot in every row.

h. T F The solution to the system $A\vec{x} = \vec{b}$ is of the form $\vec{x} = \vec{p} + t\vec{v}$ where $\vec{v}$ is any solution to the system $A\vec{x} = \vec{0}$.

i. T F A homogeneous systems of equations can be inconsistent.

j. T F Matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ is a subspace of $M_{2\times 2}$.

k. T F The function $f(x) = 0$ is a subspace of P_n.

l. T F A vector is any element of a vector space. Specifically, a polynomial is a vector because the set of all polynomials with highest degree n, P_n, is a vector space.

m. T F R^3 is a subspace of R^4.

n. T F If two spaces have the same number of basis vectors, then then are isomorphic.

o. T F The pivot rows of a matrix are always linearly independent.

p. T F The column space of an $m \times n$ matrix is a subspace of R^m.

q. T F A linear transformation defined by a 6x4 matrix can be onto, but it cannot be one-to-one.

r. T F A set of vectors are linearly independent if none of the vectors in the set are multiples of any other vector. linear combinations, too

s. T F A vector space has infinite dimensions if there is no finite basis for the space.

t. T F The kernel of a matrix is a subspace of the codomain of the matrix.

of domain

5. Determine if the following sets are subspaces. Be sure to check all the necessary conditions or find a counterexample. (5 points each)

a. $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \le 0 \right\}$.

$$\begin{bmatrix} 1 \\ -4 \end{bmatrix} + \begin{bmatrix} -5 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

not a subspace

not closed under addition

$(-4)(-3) \not\le 0$

b. The set of all odd functions, i.e. $f(-x) = -f(x)$.

it is a subspace

① $f(x)=0 = -0$ is in space

② $f(x)+g(x)$ in the space since $f(-x)+g(-x) = -f(x)-g(x)$
$= -(f(x)+g(x))$ which is still odd.

③ scalar multiples $kf(x) \Rightarrow kf(-x) = -kf(x)$ which is also still odd

c. Polynomials of the form $p(t) = (t-2)(a + bt + ct^2)$ as a subspace of P_3.

it is a subspace

① $a=b=c=0$ $p(t)=0$ in space

② $(t-2)(a+bt+ct^2) + (t-2)(d+et+ft^2) = (t-2)(a+d+(b+e)t + (c+f)t^2)$ in space

③ $k(t-2)(a+bt+ct^2) = (t-2)(ak+(bk)t+(ck)t^2)$ in space

6. Determine if the transformation $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 - 3 \\ 2x_1 - 5x_2 \end{bmatrix}$ is linear or not. If it is, prove it. If it is not, find a counterexample. (6 points)

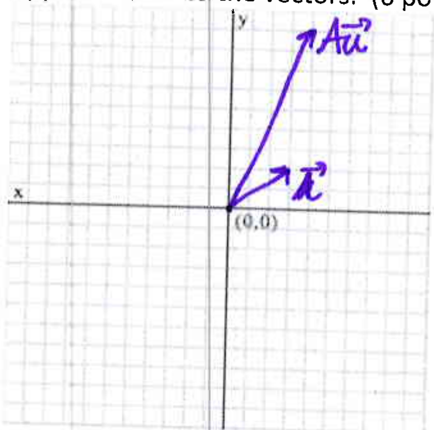
not linear.

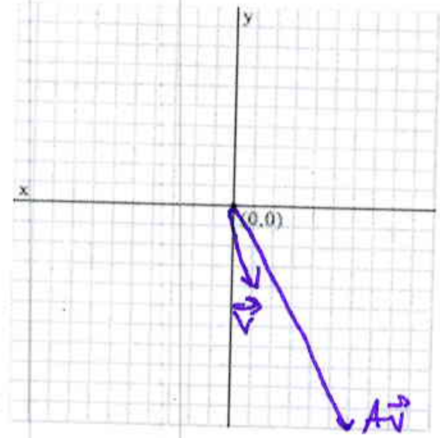
$T(\vec{0}) \neq \vec{0}$ $T\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0-2(0) \\ 0-3 \\ 2(0)-5(0) \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 0 \end{bmatrix}$

Math 2568, Exam #1, Part II, Spring 2015 Name ____________________

Instructions: Show all work. You **may** use a calculator on this portion of the exam. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question. Provide explanations where requested.

1. Consider the linear transformation matrix $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$. On the graphs below, graph the vectors $\vec{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, along with $A\vec{u}, A\vec{v}$. Describe in words what the transformation appears to do to the vectors. (6 points)





$$A\vec{u} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$A\vec{v} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 6 \\ -11 \end{bmatrix}$$

the transformation stretches the vector and skews or rotates it.

2. Find the nullspace of the system $\begin{cases} x_1 + 2x_2 - x_3 - 4x_4 + x_5 + 2x_6 = 0 \\ 4x_1 - 3x_2 \qquad\qquad\qquad + 5x_5 - x_6 = 0. \\ 2x_1 \qquad\quad - x_3 + 2x_4 \qquad\quad + 3x_6 = 0 \end{cases}$ (9 points)

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -18/5 & 13/5 & -1 \\ 0 & 1 & 0 & -24/5 & 9/5 & -1 \\ 0 & 0 & 1 & -46/5 & 26/5 & -5 \end{bmatrix}$$

x_4, x_5, x_6 free

$$\begin{aligned} x_1 &= 18/5\, x_4 - 13/5\, x_5 + x_6 \\ x_2 &= 24/5\, x_4 - 9/5\, x_5 + x_6 \\ x_3 &= 46/5\, x_4 - 26/5\, x_5 + 5x_6 \\ x_4 &= x_4 \\ x_5 &= x_5 \\ x_6 &= x_6 \end{aligned}$$

$$\Rightarrow \vec{x} = \begin{bmatrix} 18 \\ 24 \\ 46 \\ 5 \\ 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} -13 \\ -9 \\ -26 \\ 0 \\ 5 \\ 0 \end{bmatrix} s + \begin{bmatrix} 1 \\ 1 \\ 5 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

null space

3. Determine if the following sets of vectors are linearly independent. Then determine if they form a basis for the specified space. Explain your reasoning. (5 points)

a. $\left\{\begin{bmatrix}1\\5\\2\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\-1\\1\\4\end{bmatrix}\right\}, R^5$ linearly independent (2 vectors, not multiples)
not a basis for $\mathbb{R}^5$ (too few vectors)

b. $\left\{\begin{bmatrix}1\\4\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix}\right\}, R^2$ independent, is a basis
spans $\mathbb{R}^2$

c. $\left\{\begin{bmatrix}1\\-1\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\2\\1\end{bmatrix}, \begin{bmatrix}5\\0\\1\\2\end{bmatrix}, \begin{bmatrix}4\\-1\\0\\1\end{bmatrix}\right\}, R^4$ linearly independent rref to I_4
is a basis of $\mathbb{R}^4$

d. $\{1 - t^2, 3 - 2t, 5t + 7t^2\}, P_2$

$\Rightarrow \left\{\begin{bmatrix}1\\0\\-1\end{bmatrix} \begin{bmatrix}3\\-2\\0\end{bmatrix} \begin{bmatrix}0\\5\\7\end{bmatrix}\right\}$ linearly independent rref to I_3
is a basis for P_2 since P_2 is isomorphic to $\mathbb{R}^3$

e. $\{1, 1 - t, (1 - t)^2, (1 - t)^3\}, P_3$ $(1-t)^2 = 1-2t+t^2$
$(1-t)^3 = 1-3t+3t^2-t^3$

$\Rightarrow \left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \begin{bmatrix}1\\-1\\0\\0\end{bmatrix} \begin{bmatrix}1\\-2\\1\\0\end{bmatrix} \begin{bmatrix}1\\-3\\3\\-1\end{bmatrix}\right\}$ linearly independent rref to I_4
is a basis for P_3 since P_3 is isomorphic to R^4

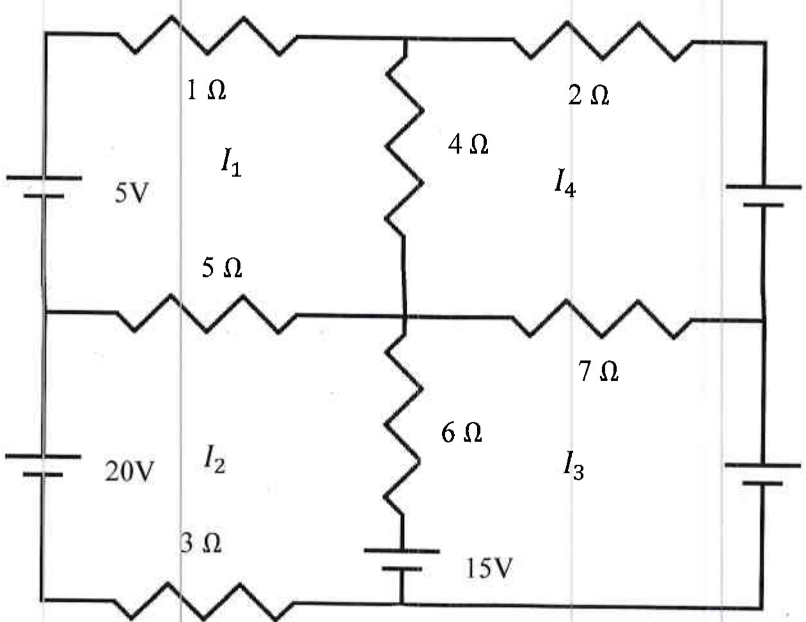
4. Suppose matrix A is a 6×8 matrix with 5 pivot columns. Determine the following. (12 points)

dim Col A = 5 dim Nul A = 3

dim Row A = 5 If Col A is a subspace of $\mathbb{R}^m$, then n = 8

Rank A = 5 If Nul A is a subspace of $\mathbb{R}^n$, then m = 6

5. Write a matrix to determine the loop currents and use your calculator to solve the system. Round your answers to two decimal places. (10 points)



$10I_1 - 5I_2 - 4I_4 = -5$

$-5I_1 + 14I_2 - 6I_3 = -20+15$

$-6I_2 + 13I_3 - 7I_4 = -15+40$

$-4I_1 \quad - 7I_3 + 13I_4 = 10$

$$\left[\begin{array}{cccc|c} 10 & -5 & 0 & -4 & -5 \\ -5 & 14 & -6 & 0 & -5 \\ 0 & -6 & 13 & -7 & 25 \\ -4 & 0 & -7 & 13 & 10 \end{array}\right]$$

$$\vec{I} = \begin{bmatrix} 3.45 \\ 3.64 \\ 6.46 \\ 5.31 \end{bmatrix}$$