

Instructions: Show all work. You may **not** use a calculator on this portion of the exam. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals). Reduce as much as possible. Be sure to complete all parts of each question. Provide explanations where requested. When you are finished with this portion of exam, get Part II.

1. An orthogonal basis for R^2 is $\left\{ \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \end{bmatrix} \right\}$. Find an expression for $\vec{x} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$ in this basis. Do not use inverse matrices (use projections). (8 points)

$$c_1 = \frac{-2 + 28}{1 + 16} = \frac{26}{17}$$

$$c_2 = \frac{-8 - 7}{1 + 16} = -\frac{15}{17}$$

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} \frac{26}{17} \\ -\frac{15}{17} \end{bmatrix}$$

2. For the matrix $A = \begin{bmatrix} -4 & 2 \\ 6 & 7 \end{bmatrix}$, find the eigenvalues and eigenvectors of the matrix. (8 points)

$$(-4 - \lambda)(7 - \lambda) - 12 = 0$$

$$\lambda^2 - 3\lambda - 28 - 12 = 0$$

$$\lambda^2 - 3\lambda - 40 = 0$$

$$(\lambda - 8)(\lambda + 5) = 0$$

$$\lambda_1 = 8, \lambda_2 = -5$$

$$\lambda_1 = 8 \Rightarrow \begin{bmatrix} -12 & 2 \\ 6 & -1 \end{bmatrix}$$

$$6x_1 = x_2$$

$$x_2 = x_2$$

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$\lambda_2 = -5 \Rightarrow \begin{bmatrix} 1 & 2 \\ 6 & 12 \end{bmatrix}$$

$$x_1 = -2x_2$$

$$x_2 = x_2$$

$$\vec{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

3. A basis for R^3 is $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} \right\}$. Use Gram-Schmidt to find an orthogonal basis for the space.

(15 points)

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix} - \frac{1-2-2+2}{1+4+4+1} \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix} + \frac{1}{10} \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 11/10 \\ -4/5 \\ 4/5 \\ 21/10 \end{bmatrix}$$

$$\times 10 \Rightarrow \begin{bmatrix} 11 \\ -8 \\ 8 \\ 21 \end{bmatrix}$$

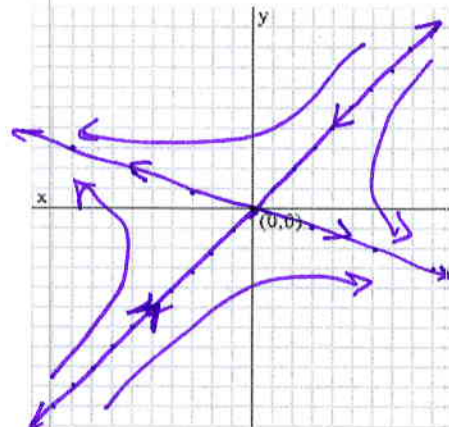
$$\vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} - \frac{1+0-4+0}{1+4+4+1} \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} - \frac{11+0+16+0}{121+64+64+441} \begin{bmatrix} 11 \\ -8 \\ 8 \\ 21 \end{bmatrix} =$$

$$\begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} + \frac{3}{10} \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} + \frac{-27}{690} \begin{bmatrix} 11 \\ -8 \\ 8 \\ 21 \end{bmatrix} = \begin{bmatrix} 20/23 \\ 21/23 \\ 25/23 \\ -12/23 \end{bmatrix} \Rightarrow \times 23 = \begin{bmatrix} 20 \\ 21 \\ 25 \\ -12 \end{bmatrix}$$

4. For each of the situations below, determine the properties of the discrete dynamical system. Is the origin an attractor, a repeller, or a saddle point? Sketch the eigenvalues on the graphs provided (if they are real) and plot some sample trajectories. (6 points each)

a. $\lambda_1 = -2.1, \lambda_2 = 0.4, \vec{v}_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Saddle point

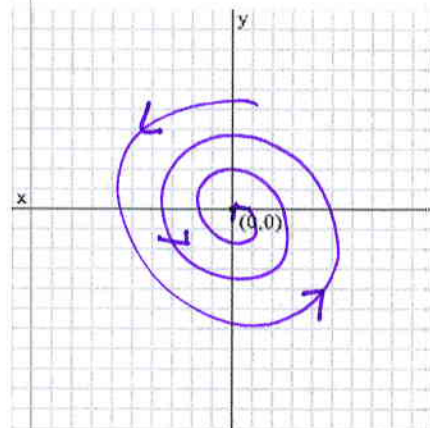


b. $\lambda_1 = 0.5 + 0.6i, \lambda_2 = 0.5 - 0.6i$.

$$\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{1}{4} + \frac{9}{25}}$$

$$= \sqrt{\frac{25 + 36}{100}} = \sqrt{\frac{61}{100}} < 1$$

Spreads in \Rightarrow attractor



5. For the vectors $\vec{u} = \begin{bmatrix} 1 \\ 9 \\ -3 \end{bmatrix}, \vec{v} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$, find the following: (4 points each)

a. $\|\vec{u}\|$

$$\sqrt{1+81+9} = \sqrt{91}$$

- b. A unit vector in the direction of \vec{u} .

$$\hat{u} = \begin{bmatrix} 1/\sqrt{91} \\ 9/\sqrt{91} \\ -3/\sqrt{91} \end{bmatrix}$$

6. Find the determinant of the matrix $\begin{bmatrix} 1 & -2 & 4 \\ 3 & 1 & -2 \\ 2 & 0 & 1 \end{bmatrix}$ by any method. (10 points)

$$2 \begin{vmatrix} -2 & 4 \\ 1 & -2 \end{vmatrix} + 1 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} =$$

$$2(\cancel{4-4}) + 1(1+6) = 7$$

7. Given that A and B are $n \times n$ matrices with $\det A = -2$ and $\det B = 7$, find the following. (4 points each)

a) $\det AB = -14$

d) $\det B^T = 7$

b) $\det A^{-1} = -\frac{1}{2}$

e) $\det 5A = 5^n(-2)$

c) $\det(-AB^3) = (-1)^n(-2)(7)^3 = (-1)^n(-686)$

8. Suppose matrix A is a 5×9 matrix with 5 pivot columns. Determine the following. (12 points)

$\dim \text{Col } A = 5$

$\dim \text{Nul } A = 4$

$\dim \text{Row } A = 5$

If $\text{Col } A$ is a subspace of \mathbb{R}^m , then $m = 5$

$\text{Rank } A = 5$

If $\text{Nul } A$ is a subspace of \mathbb{R}^n , then $n = 9$

9. Determine if the following sets are subspaces. Be sure to check all the necessary conditions or find a counterexample. (6 points each)

a. The set of complex numbers, of the form $z = a + bi$.

is a subspace

- ① $a=b=0$ 0 is in the set
- ② $(a+bi) + (c+di) = (a+c) + (b+d)i$ $a+c, b+d \in \mathbb{R}$ in set
- ③ $k(a+bi) = (ka) + (kb)i$ $ka, kb \in \mathbb{R}$ in set.

b. Polynomials of the form $p(t) = a + bt + ct^3$ as a subspace of P_3 .

is a subspace

- ① $a=b=c=0$ $0=p(t)$ in set.
- ② $(a+bt+ct^3) + (d+et+et^3) = (a+d) + (b+e)t + (c+e)t^3$ in set
- ③ $k(a+bt+ct^3) = (ka) + (kb)t + (kc)t^3$ in set.
coefficients all real.

10. Determine if the transformation $T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ -3x_2 \\ 2x_1 - 5x_2 \end{bmatrix}$ is linear or not. If it is, prove it. If it is not, find a counterexample. (6 points)

It is linear.

$$\textcircled{1} \text{ if } x_1=x_2=x_3 \quad T(0) = \begin{bmatrix} 0 - 2(0) \\ -3(0) \\ 2(0) - 5(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \checkmark$$

$$\textcircled{2} \quad T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + T \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ -3x_2 \\ 2x_1 - 5x_2 \end{bmatrix} + \begin{bmatrix} y_1 - 2y_2 \\ -3y_2 \\ 2y_1 - 5y_2 \end{bmatrix} = \begin{bmatrix} (x_1+y_1) - 2(x_2+y_2) \\ -3(x_2+y_2) \\ 2(x_1+y_1) - 5(x_2+y_2) \end{bmatrix}$$

$$T \begin{pmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{pmatrix} = \begin{bmatrix} (x_1+y_1) - 2(x_2+y_2) \\ -3(x_2+y_2) \\ 2(x_1+y_1) - 5(x_2+y_2) \end{bmatrix} \text{ check out,}$$

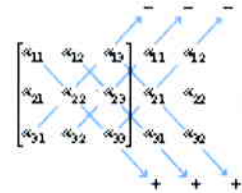
$$\textcircled{3} \quad kT \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = k \begin{bmatrix} x_1 - 2x_2 \\ -3x_2 \\ 2x_1 - 5x_2 \end{bmatrix} = \begin{bmatrix} kx_1 - 2kx_2 \\ -3kx_2 \\ 2kx_1 - 5kx_2 \end{bmatrix} \quad T \begin{pmatrix} kx_1 \\ kx_2 \\ kx_3 \end{pmatrix} = T \begin{pmatrix} kx_1 \\ kx_2 \\ kx_3 \end{pmatrix} = \begin{bmatrix} kx_1 - 2kx_2 \\ -3kx_2 \\ 2kx_1 - 5kx_2 \end{bmatrix}$$

Checks out

11. Determine if each statement is True or False. For each of the questions, assume that A is $n \times n$. (2 points each)

- a. T F If λ is an eigenvalue of A , then λ^3 is an eigenvalue of A^3 .
- b. T F If U has columns that form an orthonormal basis for the column space of U , then $U^T U = I$.
- c. T F If \vec{v} is an eigenvector of A , then \vec{v} is also an eigenvector of e^A .
- d. T F If the columns of A are linearly dependent, a least squares solution must exist.
- e. T F Row operations on a matrix do not change its eigenvalues.
- f. T F The closest point to \vec{y} to a subspace W is the orthogonal projection of the vector onto W .
- g. T F The dimension of the eigenspace of an $n \times n$ matrix is always n .
- h. T F If a vector is a linear combination of nonzero vectors from an orthogonal set, then the weights in the linear combination can be computed without matrix operations.
- i. T F Two fundamental questions about linear systems is about existence and uniqueness.
- j. T F Stochastic matrices, regardless of their size, always have at least one real eigenvector.
- k. T F The $\text{span}\{\vec{u}, \vec{v}\}$ is just the lines passing through the point \vec{u} and the origin, and the line passing through the point \vec{v} and the origin.
- l. T F In a system of ODEs, the magnitude of λ determines whether a complex eigenvalues causes the origin to repel or attract.
- m. T F The dot product or scalar product is one type of inner product.
- n. T F The solution to the system $A\vec{x} = \vec{b}$ is of the form $\vec{x} = \vec{p} + t\vec{v}$ where \vec{v} is any solution to the system $A\vec{x} = \vec{0}$.
- o. T F Matrices of the form $\begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix}$ is a subspace of $M_{2 \times 2}$.
- p. T F If two spaces have the same number of basis vectors, then then are isomorphic.
- q. T F The nullspace of an $m \times n$ matrix is a subspace of R^n .

- r. T F A linear transformation defined by a 6×4 matrix can be one-to-one, but it cannot be onto.
- s. T F A vector space has infinite dimensions if there is no finite basis for the space.
- t. T F If A has n pivots, then the system $A\vec{x} = \vec{0}$ does not have a unique solution.
- u. T F If there is a matrix C so that $CA = I$, then $\text{Rank } A = n$.
- v. T F If the dot product of two vectors is zero, then the vectors are orthogonal.
- w. T F The method of finding the determinant of a 3×3 matrix shown in the attached image cannot generalize to any other size matrix.



Instructions: Show all work. You may use a calculator on this portion of the exam. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question. Provide explanations where requested.

1. Let $W = \text{span} \left\{ \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix} \right\}$. Given $\vec{y} = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 8 \end{bmatrix}$, decompose this vector into $\vec{y}_{||}$ in W and \vec{y}_{\perp} in W^{\perp} . (12 points)

$$\frac{2+2-3+16}{1+4+1+4} \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix} + \frac{-8-1+0+24}{16+1+0+9} \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix} =$$

$$\frac{17}{10} \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix} + \frac{15}{26} \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -79/130 \\ -367/130 \\ -17/10 \\ 667/130 \end{bmatrix} = \vec{y}_{||}$$

$$\vec{y}_{\perp} = \vec{y} - \vec{y}_{||} = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 8 \end{bmatrix} - \begin{bmatrix} -79/130 \\ -367/130 \\ -17/10 \\ 667/130 \end{bmatrix} = \begin{bmatrix} 339/130 \\ 237/130 \\ 47/10 \\ 373/130 \end{bmatrix}$$

2. Use least squares to find a linear regression equation $\beta_0 + \beta_1 x = y$ for the data shown in the table below. Be sure to write the final regression equation. Round your answers to two decimal places. (12 points)

x	11	12	13	15	18	18
y	42	43	44	52	84	87

$$A = \begin{bmatrix} 1 & 11 \\ 1 & 12 \\ 1 & 13 \\ 1 & 15 \\ 1 & 18 \\ 1 & 18 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 42 \\ 43 \\ 44 \\ 52 \\ 84 \\ 87 \end{bmatrix}$$

$$(A^T A)^{-1} A^T \vec{b} = \vec{x} = \begin{bmatrix} -38.21 \\ 6.68 \end{bmatrix}$$

$$y = -38.21 + 6.68x$$

3. Use your calculator to find the equilibrium vector of the stochastic matrix $P = \begin{bmatrix} .6 & .1 & .1 & .1 \\ .1 & .6 & 0 & .3 \\ .2 & 0 & .7 & .1 \\ .1 & .3 & .2 & .5 \end{bmatrix}$.

Explain the steps you took to obtain the vector. Then demonstrate that it is the correct equilibrium vector by multiplying by P. (10 points)

$$(P - I) \vec{v} = \vec{0} \Rightarrow \begin{bmatrix} 1/5 & 1/5 & 1/5 & 1/5 \\ 34/125 & 34/125 & 34/125 & 34/125 \\ 29/125 & 29/125 & 29/125 & 29/125 \\ 37/125 & 37/125 & 37/125 & 37/125 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1/5 \\ 34/125 \\ 29/125 \\ 37/125 \end{bmatrix}$$

4. Solve the discrete dynamical system given by $\vec{x}_{k+1} = \begin{bmatrix} 0.5 & 0.4 \\ -0.6 & 1.3 \end{bmatrix} \vec{x}_k$. Find the eigenvalues and eigenvectors. Does the origin repel, attract, or is it a saddle point? (14 points)

$$(0.5 - \lambda)(1.3 - \lambda) + 0.24 = 0$$

$$.65 - 1.8\lambda + \lambda^2 + .24 = 0$$

$$\lambda^2 - 1.8\lambda + .89 = 0$$

$$\frac{1.8 \pm \sqrt{3.24 - 4(.89)}}{2} = \frac{1.8 \pm \sqrt{-.32}}{2} = \frac{1.8 \pm 2\sqrt{.08}i}{2} = .9 \pm \sqrt{.08}i$$

$$\approx .9 \pm .2828i$$

$$.9^2 + .08 = .8164 < 1$$

origin attracts. Spirals in

5. The vectors $\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ are orthogonal to each other. Let $W = \text{span}\{\vec{u}, \vec{v}\}$. Find an orthogonal basis for W^\perp . (12 points)

$$\begin{bmatrix} 2 & -3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix} \text{ rref} \Rightarrow \begin{bmatrix} 1 & 0 & 2/13 & 3/13 \\ 0 & 1 & -3/13 & 2/13 \end{bmatrix}$$

$$x_1 = -2/13 x_3 - 3/13 x_4$$

$$x_2 = 3/13 x_3 - 2/13 x_4$$

$$x_3 = x_3$$

$$x_4 = x_4$$

$$\Rightarrow \begin{bmatrix} -2/13 \\ 3/13 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 3/13 \\ -2/13 \\ 0 \\ 1 \end{bmatrix} x_4$$

$$\therefore W^\perp = \left\{ \begin{bmatrix} -2 \\ 3 \\ 13 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ 13 \end{bmatrix} \right\}$$

6. Find the nullspace of the system $\begin{cases} x_1 + 2x_2 - x_3 - 4x_4 + x_5 + 2x_6 = 0 \\ 4x_1 - 2x_2 \quad \quad \quad + 3x_5 - x_6 = 0 \\ 2x_1 \quad \quad - x_3 + 2x_4 \quad \quad + 5x_6 = 0 \end{cases}$. (12 points)

$$\begin{bmatrix} 1 & 2 & -1 & -4 & 1 & 2 \\ 4 & -2 & 0 & 0 & 3 & -1 \\ 2 & 0 & -1 & 2 & 0 & 5 \end{bmatrix} \Rightarrow \text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -2 & 4/3 & -4/3 \\ 0 & 1 & 0 & -4 & 7/6 & -13/6 \\ 0 & 0 & 1 & -6 & 8/3 & -23/3 \end{bmatrix}$$

$$x_1 = 2x_4 - 4/3 x_5 + 4/3 x_6$$

$$x_2 = 4x_4 - 7/6 x_5 + 13/6 x_6$$

$$x_3 = 6x_4 - 8/3 x_5 + 23/3 x_6$$

$$x_4 = x_4$$

$$x_5 = x_5$$

$$x_6 = x_6$$

$$\text{Null } A = \left\{ \begin{bmatrix} 2 \\ 4 \\ 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ -7 \\ -16 \\ 0 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 13 \\ 46 \\ 0 \\ 0 \\ 6 \end{bmatrix} \right\}$$

7. Determine if the following sets of vectors are linearly independent. Then determine if they form a basis for the specified space. Explain your reasoning. (6 points)

a. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}, \mathbb{R}^4$

$$\text{rref} \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

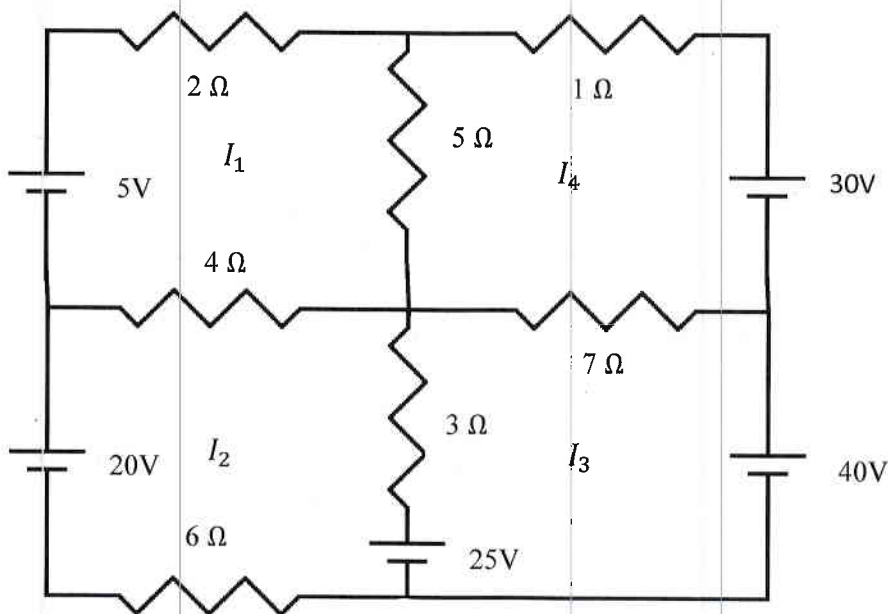
it is a basis
is independent

⊄ spans \mathbb{R}^4

b. $\{1 - t^2, 2 - 3t, 4t + t^2\}, P_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow \text{ref} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \text{is independent} \\ \text{spans } P_2 \\ \text{is a basis} \end{array}$$

8. Write a matrix to determine the loop currents and use your calculator to solve the system. Round your answers to two decimal places. (12 points)



$$\begin{aligned} 11I_1 - 4I_2 - 5I_4 &= -5 \\ -4I_1 + 13I_2 - 3I_3 &= 5 \\ -3I_2 + 10I_3 - 7I_4 &= 15 \\ -5I_1 + 7I_3 + 13I_4 &= 30 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 11 & -4 & 0 & -5 & -5 \\ -4 & 13 & -3 & 0 & 5 \\ 0 & -3 & 10 & -7 & 15 \\ -5 & 0 & -7 & 13 & 30 \end{array} \right]$$

$$\vec{I} \approx \begin{bmatrix} 5.32 \\ 4.17 \\ 9.30 \\ 9.36 \end{bmatrix}$$

9. If a basis for R^3 is $B = \left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix} \right\}$, and if a vector in the standard basis is $\vec{x} = \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}$, find its representation in the basis. (12 points)

$$P_B [\vec{x}]_B = \vec{x}$$

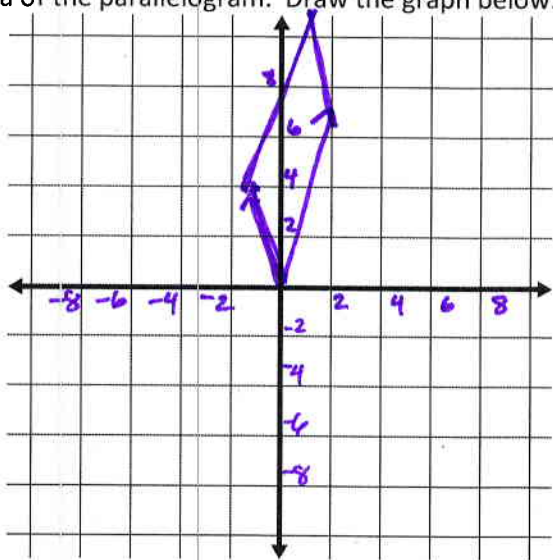
$$[\vec{x}]_B = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$P_B^{-1} \vec{x} = [\vec{x}]_B$$

$$P_B = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 0 & -3 \\ 3 & -2 & 0 \end{bmatrix}$$

$$P_B^{-1} = \begin{bmatrix} .24 & .32 & .36 \\ .36 & .48 & .04 \\ -.08 & -.44 & -.12 \end{bmatrix} = \begin{bmatrix} 6/25 & 8/25 & 9/25 \\ 9/25 & 12/25 & 1/25 \\ -2/25 & -11/25 & -3/25 \end{bmatrix}$$

10. Suppose that a parallelogram is bounded at one vertex by the vectors $\vec{u} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$. Find the area of the parallelogram. Draw the graph below. (10 points)



$$\det \begin{bmatrix} 2 & -1 \\ 7 & 4 \end{bmatrix} = 8 + 7 = 15$$