Math 2568, Quiz #10, Spring 2015 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Is the vector $\vec{x} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ and eigenvector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 2 & 7 \\ 5 & 6 & 4 \end{bmatrix}$? If it is, state the eigenvalue.

$$\begin{bmatrix} 3 & 6 & 7 \\ 3 & 2 & 7 \\ 5 & 6 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3-12+14 \\ 3-4+14 \\ 5-12+8 \end{bmatrix} = \begin{bmatrix} 1 \\ 13 \\ 1 \end{bmatrix}$$

no, it is not an eigenvector $\begin{bmatrix} 1 \\ 13 \\ 1 \end{bmatrix}$ is not a multiple of $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

2. Is $\lambda = 1$ an eigenvalue of $\begin{bmatrix} 4 & -2 & 3 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If so, find the corresponding eigenvector.

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 2 & 1 \\ -3 & 4 & 4 \end{bmatrix} \text{ rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

no, $\lambda=1$ is not an eigenvalue of the matrix. if it was $A-\lambda I$ would be dependent.

3. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -1 \\ 6 & -4 \end{bmatrix}$.

$$(1-\lambda)(-4-\lambda)+6=0$$

$$-4-\lambda+4\lambda+\lambda^2+6=0$$

$$\lambda^2+3\lambda+2=0$$

$$(\lambda+1)(\lambda+2)=0$$

$$\lambda_1=-1, \quad \lambda_2=-2$$

$$\begin{bmatrix} 2 & -1 \\ 6 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$$

$-3R_1+R_2 \to R_2$

$2x_1 - x_2 = 0$ $\quad x_2$ free

$2x_1 = x_2$

$x_2 = x_2$

or $\quad x_1 = \frac{1}{2}x_2 \quad \Rightarrow \vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$x_2 = x_2$

if $x_2 = 2$

$$\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}$$

$-2R_1+R_2 \to R_2$

$3x_1 - x_2 = 0$ $\quad x_2$ free

$3x_1 = x_2$

$x_2 = x_2$

or $\quad x_1 = \frac{1}{3}x_2 \quad \Rightarrow \vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$x_2 = x_2$

if $x_2 = 3$