Math 2568, Quiz #12, Spring 2015 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the equilibrium vector of the matrix $P = \begin{bmatrix} .1 & .5 \\ .9 & .5 \end{bmatrix}$ algebraically. Be sure to properly normalize the vector.

$P - I = \begin{bmatrix} -.9 & .5 \\ .9 & -.5 \end{bmatrix}$ $\quad .9x_1 = .5x_2$, $x_2 = x_2$ $\Rightarrow$ $x_1 = \frac{.5}{.9}x_2$, $x_2 = x_2$ $\Rightarrow$ $x_1 = \frac{5}{9}x_2$, $x_2 = x_2$

if $x_2 = 9 \Rightarrow \vec{v} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$ $\quad 5 + 9 = 14 \Rightarrow$

$q = \begin{bmatrix} 5/14 \\ 9/14 \end{bmatrix}$

2. Use your calculator to find the equilibrium vector of the stochastic matrix $P = \begin{bmatrix} .7 & .1 & .1 \\ .2 & .8 & .2 \\ .1 & .1 & .7 \end{bmatrix}$.

Explain the steps you took to obtain the vector. Then demonstrate that it is the correct equilibrium vector by multiplying by P.

$P^\wedge 60 = \begin{bmatrix} 1/4 & 1/4 & 1/4 \\ 1/2 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/4 \end{bmatrix}$ as $P^k \to P^\infty$ the columns converge on q.

$\therefore \vec{q} = \begin{bmatrix} 1/4 \\ 1/2 \\ 1/4 \end{bmatrix}$ or $\begin{bmatrix} .25 \\ .5 \\ .25 \end{bmatrix}$

3. Solve the discrete dynamical system given by $\vec{x}_{k+1} = \begin{bmatrix} 0.3 & 0.4 \\ -0.3 & 1.1 \end{bmatrix} \vec{x}_k$. Sketch a graph of the eigenvectors and plot some sample trajectories. Is the origin an attractor, a repeller or a saddle point?

$(.3-\lambda)(1.1-\lambda)+.12=0$

$.33-1.4\lambda+\lambda^2+.12=0$

$\lambda^2-1.4\lambda+.45=0$

$(\lambda-.9)(\lambda-.5)=0$

$\lambda=.9,\ \lambda=.5$

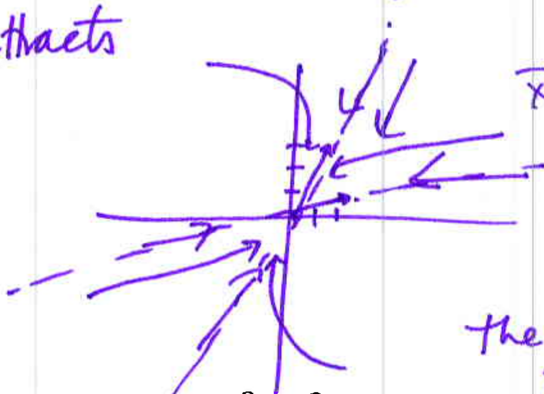
the origin attracts

$\lambda_1 \Rightarrow \begin{bmatrix} -.6 & .4 \\ -.3 & 0.2 \end{bmatrix}$ $\quad -.3x_1=-.2x_2$, $x_1=\frac{2}{3}x_2$, $x_2=x_2$

$\vec{x}_1=\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\lambda_2 \Rightarrow \begin{bmatrix} -.2 & .4 \\ -.3 & .6 \end{bmatrix}$ $\quad -.2x_1=-.4x_2$, $x_1=2x_2$, $x_2=x_2$

$\vec{x}_2=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$



the vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ corresponding to larger λ persists longer.

4. Solve the system of ODEs given by $\vec{x}' = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \vec{x}$. Sketch a graph of the eigenvectors and plot some sample trajectories. Is the origin an attractor, a repeller or a saddle point?

$(2-\lambda)(-2-\lambda)+3=0$

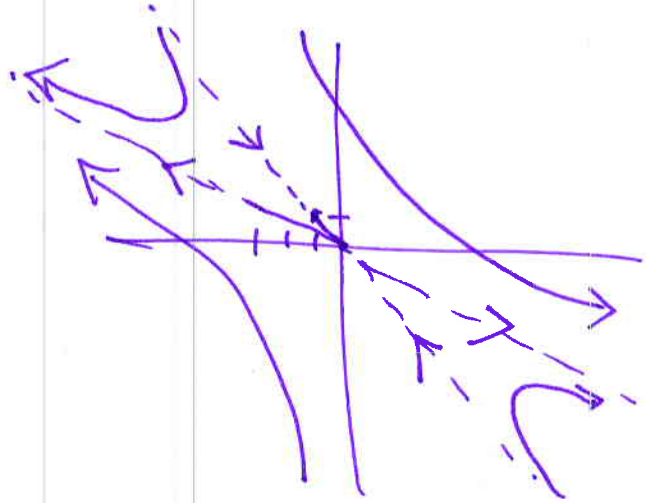
$-4+\lambda^2+3=0$

$\lambda^2-1=0$

$\lambda=\pm 1$ Saddlepoint

$\lambda=1 \Rightarrow \begin{bmatrix} 1 & 3 \\ -1 & -3 \end{bmatrix}$ $\quad x_1=-3x_2$, $x_2=x_2$ $\quad \vec{v}_1=\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

$\lambda=-1 \Rightarrow \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$ $\quad +x_1=\mp x_2$, $x_2=x_2$ $\quad \vec{v}_2=\begin{bmatrix} -1 \\ 1 \end{bmatrix}$



$\vec{x}=c_1\begin{pmatrix} -3 \\ 1 \end{pmatrix}e^{t}+c_2\begin{pmatrix} -1 \\ 1 \end{pmatrix}e^{-t}$

expands $\qquad$ Collapses