Math 2568, Quiz #3, Spring 2015 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Solve the system of equations $\begin{cases} x_1 + 2x_2 - 3x_3 = 0 \\ 2x_1 + x_2 - 3x_3 = 0 \\ -x_1 + x_2 \qquad = 0 \end{cases}$ and write the solution in parametric form.

rowreduce (rref) on coeff A $\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 - x_3 = 0$, $x_2 - x_3 = 0$, x_3 is free $\Rightarrow$

$$\begin{aligned} x_1 &= x_3 \\ x_2 &= x_3 \\ x_3 &= x_3 \end{aligned} \quad \Rightarrow \quad \vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3 \text{ or } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$$

2. Determine if the following sets are subspaces. Be sure to check all the necessary conditions or find a counterexample.

a. $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} | xy \geq 0 \right\}$. not a subspace

for $xy \geq 0$ then both $x, y \geq 0$ or $x, y \leq 0$

for addition consider the vectors $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ -1 \end{bmatrix}$ both are in V. but $\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} -4 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ where the product of $(-3)(2)$ are not ≥ 0. not closed under addition.

b. $H = \left\{ \begin{bmatrix} 3s + 4t \\ s \\ 2s - 3t \\ 5t \end{bmatrix}, s, t \in R \right\}$ $\Rightarrow H = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 0 \end{bmatrix} s + \begin{bmatrix} 4 \\ 0 \\ -3 \\ 5 \end{bmatrix} t$ is a subspace

1. if $s = t = 0$ then $\vec{0}$ is in the space.

2. $\begin{bmatrix} 3s+4t \\ s \\ 2s-3t \\ 5t \end{bmatrix} + \begin{bmatrix} 3a+4b \\ a \\ 2a-3b \\ 5b \end{bmatrix} = \begin{bmatrix} 3(s+a)+4(t+b) \\ (s+a) \\ 2(s+a)-3(t+b) \\ 5(t+b) \end{bmatrix}$ which is a vector of the correct form w/ $s+a=u$, $t+b=w$ and both real. closed under addition

3. $k \begin{bmatrix} 3s+4t \\ s \\ 2s-3t \\ 5t \end{bmatrix} = \begin{bmatrix} 3(ks)+4(kt) \\ ks \\ 2(sk)-3(kt) \\ 5(kt) \end{bmatrix}$ where ks, and kt are real #'s closed under scalar multiplication

c. The set of all complex numbers. $\mathbb{C} = a + bi$ where a, b are real.

① if $a, b = 0$ then $\vec{0}$ in space.

② $(a+bi)+(c+di) = (a+c)+(b+d)i$ where $a+c$ is real, as is $(b+d)$

closed under addition.

③ $k(a+bi) = (ka)+(kb)i$ where ka, and kb are both real.

closed under scalar multiplication

is a subspace (isomorphic to $\mathbb{R}^2$)

d. Polynomials of the form $p(t) = 1 + bt + ct^2$.

not a subspace

fails all three tests

for instance no $\vec{0}$ is in the set.

if $b=c=0$ $p(t) = 1 \neq 0$

note: The value of t is not relevant here or ever.

The properties must be true for all values of t