Math 2568, Quiz #4, Spring 2015 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. The matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 3 & 0 \end{bmatrix}$ is a linear transformation, $T(\vec{x}) = A\vec{x}$.
   a. What is the domain of A?

   $\mathbb{R}^3$

   b. What is the codomain of A?

   $\mathbb{R}^2$

   c. Is the linear transformation one-to-one?

   no

   d. Is the linear transformation onto?

   yes

2. Give an example of a linear transformation matrix with the specified properties.
   a. A rotation matrix that rotates vectors in R^2 by $\theta = \frac{\pi}{4}$.

   $$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

   b. A matrix that scales the x_1 direction by a factor of 2, and scales and reflects the x_2 direction by a factor of three.

   $$\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

c. A linear transformation that maps $\vec{e}_1$ onto $\begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$, maps $\vec{e}_2$ onto $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, and maps $\vec{e}_3$ onto $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$.

$$\begin{bmatrix} 4 & 2 & 1 \\ -1 & 1 & 0 \\ 5 & 0 & -2 \end{bmatrix}$$

d. A projection transformation in R^3 that maps onto a two-dimensional subspace.

answers will vary → any 3x3 matrix w/ only 2 pivots works

e.g. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. Determine if the transformation $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2|x_2| \\ x_1 - 4x_2 \end{bmatrix}$ is linear or not. If it is, prove it. If it is not, find a counterexample.

it is not.

Consider $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1+2(1) \\ 1-4(1) \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1+2(1) \\ 1-4(-1) \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) + T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -3 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2+2(0) \\ 2-4(0) \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 6 \\ 2 \end{bmatrix} \neq \begin{bmatrix} 2 \\ -2 \end{bmatrix}$