Math 2568, Quiz #8, Spring 2015 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. List 4 properties of the Invertible Matrix Theorem.

answers will vary – A is $n \times n$ then the following are equivalent.

A is invertible

A^T is invertible

A has n pivots

A reduces to $n \times n$ identity.

Nul $A = \{\vec{0}\}$

Col A form as basis for $\mathbb{R}^n$

etc.

2. If a basis for R^3 is $B = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix} \right\}$, and given $[\vec{x}]_B = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, find $\vec{x}$ in the standard basis.

$$1\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix} + (-2)\begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0-8 \\ -2+0+6 \\ 3+0+0 \end{bmatrix} = \begin{bmatrix} -7 \\ 4 \\ 3 \end{bmatrix} = \vec{X}$$

$$P_B [\vec{x}]_B = \vec{X}$$

3. If a vector in the standard basis is $\vec{x} = \begin{bmatrix} 1 \\ -7 \\ 8 \end{bmatrix}$, find its representation in the basis in problem #2.

$$P_B = \begin{bmatrix} 1 & 5 & 4 \\ -2 & 0 & -3 \\ 3 & -2 & 0 \end{bmatrix}$$

$$P_B^{-1} = \begin{bmatrix} 6/35 & 8/35 & 3/7 \\ 9/35 & 12/35 & 1/7 \\ -4/35 & -17/35 & -2/7 \end{bmatrix}$$

$$P_B^{-1} \vec{x} = [\vec{x}]_B = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

4. Consider the basis $C = \left\{ \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \right\}$, and the vector $[\vec{x}]_C = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$. Find the representation of the vector in the basis B in problem #2.

$$P_C [\vec{x}]_C = P_B [\vec{x}]_B \qquad P_C = \begin{bmatrix} 1 & -3 & 2 \\ -1 & 4 & -2 \\ -3 & 9 & 4 \end{bmatrix}$$

$$P_B^{-1} P_C [\vec{x}]_C = [\vec{x}]_B$$

$$\begin{bmatrix} -47/35 & 149/35 & 8/5 \\ -18/35 & 66/35 & 2/5 \\ 43/35 & -146/35 & -2/5 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 606/35 \\ 244/35 \\ -509/35 \end{bmatrix}$$