

MAT 135, Discussion Questions 3.09

1. Consider a state license plate with six characters total, with three capital letters (not including O), followed by three numbers (i.e. ABC 123). How many different plates are possible?

$$55^3 \cdot 10^3 = 1,562,500$$

2. How many of the plates above contain only vowels and even numbers? What is the probability that a randomly chosen plate in this state contains only vowels and even numbers?

$$4^3 \cdot 5^3 = 8000$$

(no O)

$$\frac{8000}{1,562,500} = 5.12 \times 10^{-4}$$

3. Suppose that a wardrobe has three pairs of pants, five shirts, two belts, six pairs of socks, three blazers, and two pairs of shoes. How many different outfits are possible if you need to choose one of each?

$$3 * 5 * 2 * 6 * 3 * 2 = 1080$$

4. How many ways can you arrange a deck of cards? <http://ed.ted.com/lessons/how-many-ways-can-you-arrange-a-deck-of-cards-yannay-khaikin#review>

$$52! = 8.066 \times 10^{67} \quad (\text{a lot!})$$

5. What is the value of each of these expressions containing factorials?

a. $5! = 120$

c. $0! = 1$

d. $9! = 362,880$

b. $\frac{9!}{3!} = 60,480$

e. $\frac{10!}{5!5!} = 252$

6. When we are counting things, what two conditions need to be satisfied to use combinations?

no repetitions, order doesn't matter

7. When we are counting things, what two conditions need to be satisfied to use permutations?

no repetitions, order does matter

8. Give a situation where you would use combinations.

groups, committees, couples, hands of cards

9. Give a situation where you would use permutations.

Batting line up, standing in line, etc.

10. Give three notations for combinations, and three notations for permutations.

nCr , $C(n,r)$, $\binom{n}{r}$
 C_r^n etc.

$P(n,r)$, nPr , ${}^n P_r$, etc.

11. How does one read $\binom{10}{3}$? What is its value?

"10 choose 3" = 120

12. Suppose you want to rearrange the letters of 'caroline' to make a puzzle. How many different sequences of these letters are possible?

$8! = 40,320$ (no repeated letters)

13. There are four prizes of differing values in a raffle drawing with 100 tickets sold. How many different ways can you distribute the four prizes?

$100P4 = 94,109,400$

14. A math club has 15 members and they want to form a three-member committee to plan an event. In how many different ways can the committee be chosen?

$\binom{15}{3} = 455$

15. A coin is flipped 12 times. In how many different ways can the coin come up heads 5 times?

$${}^{12}C_5 = 792$$

16. Sometimes it's necessary not just to count the number of permutations and combinations, but to actually list them all. You might find this site helpful.
<http://www.mathsisfun.com/combinatorics/combinations-permutations-calculator.html> Choose a list of elements and a certain size subset of them, and either permutations and combinations. Copy or print the list of elements it produces.

Answers will vary