MAT 201, Quiz #13, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the velocity, acceleration and jerk of a particle moving along path $\vec{r}(t) = t^2\hat{\imath} + 2t\hat{\jmath} + \ln t\,\hat{k}$, $(t > 0)$ at $t = 1$.

$v(t) = r'(t) = 2t\hat{\imath} + 2\hat{\jmath} + \frac{1}{t}\hat{k}$ $\qquad v(1) = 2\hat{\imath} + 2\hat{\jmath} + 1\hat{k}$

$a(t) = r''(t) = 2\hat{\imath} + 0\hat{\jmath} - \frac{1}{t^2}\hat{k}$ $\qquad a(1) = 2\hat{\imath} - 1\hat{k}$

$j(t) = r'''(t) = 0\hat{\imath} + 0\hat{\jmath} + \frac{2}{t^3}\hat{k}$ $\qquad j(1) = 2\hat{k}$

2. Given $\vec{a}(t) = 2\hat{\imath} + 6t\hat{\jmath} + 12t^2\hat{k}, \vec{v}(0) = \hat{\imath}, \vec{r}(0) = \hat{\jmath} - \hat{k}$, find the position vector.

$v(t) = \int a(t) = (2t + C_1)\hat{\imath} + (3t^2 + C_2)\hat{\jmath} + (4t^3 + C_3)\hat{k}$ $\qquad v(t) = (2t+1)\hat{\imath} + 3t^2\hat{\jmath} + 4t^3\hat{k}$

$v(0) = \hat{\imath}$ $\qquad C_1 = 1,\ C_2 = 0,\ C_3 = 0$

$r(t) = \int v(t) = (t^2 + t + C_1)\hat{\imath} + (t^3 + C_2)\hat{\jmath} + (t^4 + C_3)\hat{k}$

$C_2 = 1$, $C_3 = -1$

$C_1 = 0$ $\qquad C_2 = 1$ $\qquad C_3 = -1$

$$\boxed{r(t) = (t^2 + t)\hat{\imath} + (t^3 + 1)\hat{\jmath} + (t^4 - 1)\hat{k}}$$

3. Use Lagrange multipliers to maximize the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x + y + z = 12$.

$x + y + z - 12 = 0$

$\nabla f = 2x\hat{\imath} + 2y\hat{\jmath} + 2z\hat{k}$

$\nabla g = \hat{\imath} + \hat{\jmath} + \hat{k}$ $\qquad \lambda \nabla g = \lambda\hat{\imath} + \lambda\hat{\jmath} + \lambda\hat{k}$

$2x = \lambda,\ 2y = \lambda,\ 2z = \lambda$

$x = \lambda/2 \quad y = \lambda/2 \quad z = \lambda/2$

$x + y + z = 12$

$\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{2} = 12$

$3\lambda = 24$

$\lambda = 8$

$x = \frac{8}{2} = 4,\ y = 4,\ z = 4$

extrema at

$(4, 4, 4)$