MAT 201, Quiz #4, Spring 2016   Name _____KEY_____

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the limit.

a. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y e^y}{x^4 + 4y^2}$

let $y = kx^2$

$\lim\limits_{x\to 0} \dfrac{x^2 \, kx^2 e^{kx^2}}{x^4 + 4(kx^2)^2} = \lim\limits_{x\to 0} \dfrac{kx^4 e^{kx^2}}{x^4(1+4k^2)}$

$= \lim\limits_{x\to 0} \dfrac{k e^{kx^2}}{1+4k^2} = \dfrac{k}{1+4k^2}$

DNE
Since value of limit depends on k

b. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 4y^2}{x^2 + 2y^2}$

let $y = kx$

$\lim\limits_{x\to 0} \dfrac{x^4 - 4(kx)^2}{x^2 + 2(kx)^2} =$

$\lim\limits_{x\to 0} \dfrac{x^4 - 4k^2 x^2}{x^2 + 2k^2 x^2} =$

$\lim\limits_{x\to 0} \dfrac{x^2(x^2 - 4k^2)}{x^2(1+2k^2)} =$

$\dfrac{-4k^2}{1+2k^2}$ **DNE**

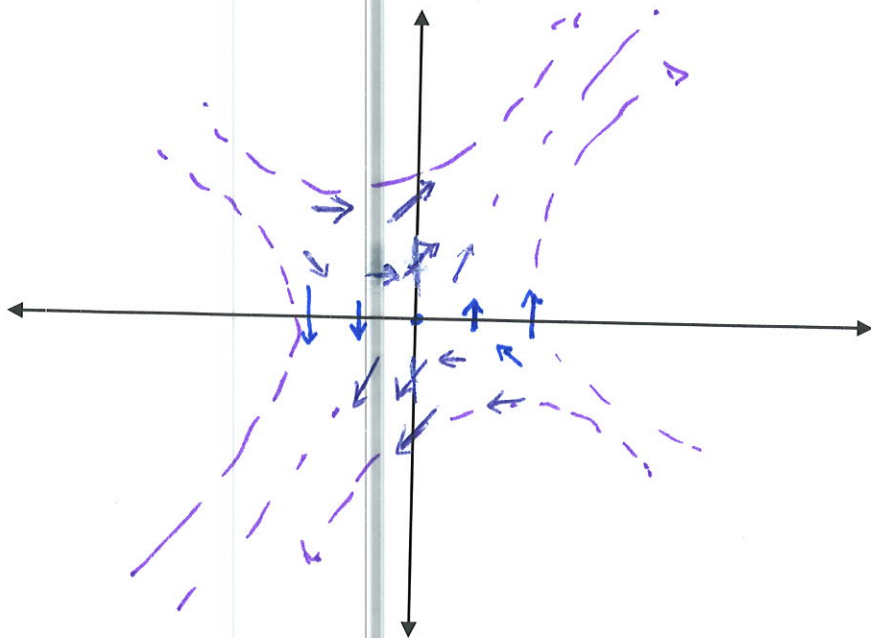c. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy + yz}{x^2 + y^2 + z^2}$ in spherical

$\lim\limits_{\rho\to 0} \dfrac{\rho\sin\varphi\cos\theta \, \rho\sin\varphi\sin\theta + \rho\sin\varphi\sin\theta \, \rho\cos\varphi}{\rho^2}$

$= \lim\limits_{\rho\to 0} \dfrac{\rho^2(\sin^2\varphi\cos\theta\sin\theta + \sin\varphi\sin\theta\cos\varphi)}{\rho^2}$

DNE
Since limit depends on φ & θ

2. Sketch the vector field
$\vec{F}(x,y) = y\hat{\imath} + (x+y)\hat{\jmath}$.
Plot enough points to see the general behavior of the field.

x	y	$\hat{\imath}$	$\hat{\jmath}$
0	0	0	0
1	0	0	1
0	1	1	1
-1	0	0	-1
0	-1	-1	-1
1	1	1	2
1	-1	-1	0
-1	1	1	0
-1	-1	-1	-2
0	2	2	2
0	-2	-2	-2
-2	1	1	-1
2	-1	-1	1

3. Use the sketch of the vector field below to sketch at least 5 level curves (recall that the level curves are perpendicular to the direction of the field). If a surface was defined by this field, in which direction would a particular starting at the indicated point roll?

level curves

particle rolls here