MAT 212, Quiz #13, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the general solution for each system and describe the behavior as $t \to \infty$.

   a. $\vec{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \vec{x}$.

$(3-\lambda)(-1-\lambda) + 8 = 0 \quad \lambda^2 - 2\lambda + 5 = 0$

$\lambda = \frac{2 \pm \sqrt{-16}}{2} = 1 \pm 2i$

$\begin{pmatrix} 3-1-2i & -2 \\ 4 & -1-1-2i \end{pmatrix} = \begin{pmatrix} 2-2i & -2 \\ 4 & -2-2i \end{pmatrix}$ $\begin{matrix} 2 & -1-i \end{matrix}$ $\quad 2x_1 = \frac{(1+i)}{2} x_2 \quad x_2 = x_2$

$e^t \begin{pmatrix} 1+i \\ 2 \end{pmatrix} (\cos 2t + i \sin 2t) =$

$e^t \begin{bmatrix} \cos 2t + i\cos 2t + i \sin 2t - \sin 2t \\ 2\cos 2t + 2i \sin 2t \end{bmatrix}$

$\vec{x} = c_1 e^t \begin{bmatrix} \cos 2t - \sin 2t \\ 2\cos 2t \end{bmatrix} + c_2 e^t \begin{bmatrix} \cos 2t + \sin 2t \\ 2 \sin 2t \end{bmatrix}$

Spirals outward

   b. $\vec{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \vec{x}$

$(3-\lambda)(-1-\lambda) + 4 =$

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0$

$\lambda = 1$

$\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix} \Rightarrow \begin{matrix} x_1 = 2x_2 \\ x_2 = x_2 \end{matrix} \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\left[\begin{matrix} 2 & -4 \\ 1 & -2 \end{matrix} \middle| \begin{matrix} 2 \\ 1 \end{matrix}\right] \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\vec{x}(t) = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \left[t e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right]$

$\vec{x} \to \infty$ as $t \to \infty$

2. Find the fundamental matrix for the system $\vec{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \vec{x}$.

$(3-\lambda)(-2-\lambda) + 4$

$\lambda^2 - \lambda - 2 = 0$

$(\lambda - 2)(\lambda + 1) = 0$

$\lambda = 2, \lambda = -1$

$\begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix} \qquad \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}$

$\begin{matrix} x_1 = 2x_2 \\ x_2 = x_2 \end{matrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{2t} \qquad \begin{matrix} 2x_1 = \frac{x_2}{2} \\ x_2 = x_2 \end{matrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t}$

$\Psi = \begin{pmatrix} 2e^{2t} & e^{-t} \\ e^{2t} & 2e^{-t} \end{pmatrix}$

3. Use Euler's method on $y' = 2te^{-ty}, y(0) = 1$ to find $y(1)$ in 100 steps. Complete the first three steps of the calculation with at least 6 decimal places.

$\frac{1}{100} = .01$

$y_0 = 1 \quad t_0 = 0 \qquad m_1 = 2(0)e^{-0(1)} = 0 \qquad y_1 = 0(.01) + y_0 = 1$

$y_1 = 1 \quad t_1 = .01 \qquad m_2 = 2(.01)e^{-.01(1)} = .0198\ldots \qquad y_2 = .0198(.01) + 1 = 1.00019801$

$y_2 = 1.00019801 \quad t_2 = .02 \qquad m_3 = 2(.02)e^{-.02(1.00019801)}$

$y_3 = .0392077917(.01) + 1.00019801 = 1.000590088$

$\boxed{y_3 = 1.000590088} \quad t_3 = .03$